The M. Em-Power Principle and The M. Ordinality Principle: Two Com-possible Approaches, not always Equivalent

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ABSTRACT

The present paper, after having synthetically recalled the fundamental steps that led us to the formulation of both the Maximum Em-Power Principle and the Maximum Ordinality Principle, and after having pointed out their corresponding and proper characteristics, it will show their logical, theoretical and operative com-possibility of adoption, even if with some differences, in particular under transient conditions. This is why they result as being not always "equivalent".

INTRODUCTION. HISTORICAL PRESUPPOSITIONS OF THE MATHEMATICAL FORMULATION OF THE MAXIMUM EM-POWER PRINCIPLE

I had a preliminary idea of the new physical quantity termed as "Emergy" in 1993, by reading a scientific paper prepared for an official publication[1], sent to me by Prof. Sergio Ulgiati.

During the following months I had several email contacts with Prof. Sergio Ulgiati (first author of the paper), whom I continuously asked for additional documents to better understand the *logical and physical origin* of such a new physical concept, together with its rigorous definition. In this respect, I want to thank Prof. Ulgiati for his willingness to deal with such a fastidious interlocutor.

What first impressed me was the special Algebra adopted (termed as "*Emergy Algebra*"), which was very different from the traditional one, but at the same time so "similar" to the *Differential Fractional Calculus* I had begun to study six years before. In fact, the first time I met Prof. Ulgiati, I did not miss the opportunity of asking him: "Why don't you adopt Fractional Calculus in Emergy Analysis?".

Some weeks later Prof. Ulgiati invited me to attend a special course on Emergy Analysis given by Prof. Mark T. Brown in Siena (September 1993), during which I had the opportunity of meeting another extremely important person in my life. My collaboration with Prof. Brown, in fact, started precisely there, in Siena: together we prepared a general scheme which reproduced, by means of the special *System Diagrams* adopted in Emergy Analysis, the Italian Energy Supply System in terms of Thermodynamics, Economics and Rights (Norms and Laws), which was successively presented at the International Workshop of Porto Venere (Giannantoni C., 1998).

I must also thank Prof. Brown in a particular way because he encouraged my attempts at introducing the "*mathematically Equivalent Source Terms*" in Emergy Algebra, which represented one of the fundamental steps for the successive mathematical formulation of the Maximum Em-Power Principle.

In May 24, 1995 I personally met Prof. Howard T. Odum, who had been invited by our ENEA's Division to give two splendid Lectures at ENEA's Headquarters. On that occasion Prof. Odum was so kind as to spend a whole day with me (in the presence of his Lady Prof. Elisabeth Odum and Prof. Ulgiati), answering my questions, dissolving my doubts, and suggesting possible new lines of research. I cannot fail to mention that, among other things, he also gave me five of his most famous books as a present.

At the end of that day, after experiencing such a profound willingness shown by Prof. Odum, I frankly expressed my perplexity about the Maximum Em-Power Principle when termed as "Thermodynamic" Principle. In fact,

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[1] Ulgiati S., Odum H. T., Bastianoni S. *Emergy use, environmental loading and sustainability. An emergy analysis of Italy.* Ecological Modeling 73 (1994) 215-268 (received 9 December 1992, accepted 10 August 1993).

I observed, without a general mathematical formulation of such a Principle it is rather difficult to decide whether the M. Em-P. Principle is a "Thermodynamic" Principle or not.

Prof. Odum, with a sweet and delicate smile, promptly answered: "You should do it". "In fact I agree with you, and I believe you can succeed in this task". That hope, so openly manifested by Prof. Odum, indelibly marked the "official" birth of my research for a Mathematical Formulation of the Maximum Em-Power Principle.

For years, in fact, his paternal look encouraged me in this difficult task. His faithful invitation sustained me especially when the major difficulties arose, during the successive stages of the formulation.

Thank you Prof. Odum. Such a Mathematical Formulation is more your merit than mine. Without your sweet smile and paternal trusting invitation, I would have never found the courage to face such a hard task.

1. MATHEMATICAL FORMULATION OF THE MAXIMUM EM-POWER PRINCIPLE

The Mathematical Formulation of the Maximum Em-Power Principle was given in 2001, and it was contextually presented at the 2nd Biennial Emergy Conference (September 20-22, 2001), (Giannantoni C., 2001b).

The mathematical formulation is here synthetically reproduced

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \to Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (1),$$

which faithfully translates in formal terms its corresponding verbal enunciation: "*Every System tends to organize its internal structure to generate progressively increasing spring-Emergy levels in order to maximize the flow of processed* (*or "useful"*) *Emergy*".

In Eq. (1) in fact:

φ_v^* = the "*equivalent*" Source Term per unit volume

Γ = the *local* structural amplification and re-normalization factor (corresponding to the product of the coefficients γ_k^* and γ_k in the case of discrete form), which also accounts for the structural variations with time;

$$em_v^* = em_{v,m} + em_{v,q}^+ + em_{v,w}^+ \qquad (2)$$

where

$em_{v,m} = C \cdot \rho \cdot ex$ is the Emergy per unit volume associated to the *mass*

(thus transportable by mass flows)

$em_{v,q}^+$ = the Emergy per unit volume associated to *heat* source terms

$em_{v,w}^+$ = the Emergy per unit volume associated to *work* source terms.

In this way, the formulation is valid for *any Domain* (D^*) belonging to *Universal Space* ($S_U(t)$).

Additional details about such a formulation can be found in the paper presented at the 2nd Biennial Emergy Conference (Giannantoni C. 2001b), and, even more, in the book titled "The Maximum Em-Power Principle as the basis for Thermodynamics of Quality (Ginnantoni C., 2002).

In fact, at the end of the 2nd Emergy Conference, Prof. Odum, who had appreciated such a formulation, invited me to write book, that was effectively published the year after (2002).

In the preface of the book, Prof. Mark Brown, after having pointed out the basic characteristics of the book, explicitly mention two aspects: a) "The Center for Environmental Policy, through a generous gift from H.T. Odum has made funds available to help with the book's publication"; b) "Unfortunately, Odum, died on September 11, 2002, before

the publication of this book. During his last months, we discussed this work, and his feeling was that it represents a major contribution to the field, and that it was a very important next step in the formulation of his life work."

2. THE FUNDAMENTAL RESULTS PRESENTED IN THE BOOK ON THE M. EM-P. P.

The Fundamental results presented in the book can be synthesized as follows:

- The *First Principle of Thermodynamics*, *The Second Principle of Thermodynamics*, *The Third Principle of Thermodynamics*, together with *The Minimum Action Principle* have been reconsidered in the light of The Maximum Em-Power Principle

- such a Process has clearly shown that all these Principles can be "obtained" by The Maximum Em-Power Principle. However, not by "*deduction*", but only by "*reduction*".

These results confirm, sustain and reinforce Odum's assertion that The Maximum Em-Power Principle has to be considered as The Fourth Thermodynamic Principle. However, not in terms of *quantity*, but in terms of *Quality*.

3. SUBSEQUENT RESEARCHES ON THE M. EM-P. P. IN THE YEARS 2003-2009

All the researches pertaining to this period were essentially devoted to show the *Relevance* of The Maximum Em-Power Principle in *several scientific fields*.

All the correlative results, apart those presented at the various Biennial Emergy Conferences, where synthesized in two books (in Italian), titled "*The Lightness of Quality*" (Giannantoni C., 2007) and "*The Ascendency of Quality*" (Giannantoni C., 2008b), respectively.

The first book presented the novelties introduced by The Maximum Em-Power Principle in the various Scientific Felds listed at column 1 of Tab. 1, while the second book presented the correlative novelties in the fields which are listed at column 2

Table 1

The Lightness of Quality	The Ascendency of Quality
Classical Mechanics **Relativistic Mechanics** **Quantum Mechanics**	**Electromagnetism** **Inorganic Chemistry** **Organic Chemistry** **Biological Chemistry** **Molecular Biology** **Ontogenetic Biology** **Phylogenetic Biology** **Cosmology**

At the same time, and in parallel to the abovementioned lines of research, a particular attention was persistently devoted to a possible improvement of the *formal language* adopted in the M. Em-P. P., as it will be illustrated in the next paragraph.

4. THE RESEARCH FOR A MORE APPROPRIATE FORMAL LANGUAGE IN THE CONTEXT OF THE M.EM-P.P.

Such a research was mainly focused on the cases in which The Maximum Em-Power Principle is adopted to describe a given System *under transient conditions* (Giannantoni C., 2004a,b, 2006, 2008a).

This is because Emergy is an "Emerging Property" of Self-Organizing Systems. Consequently, when the System is analyzed under steady state conditions, the Rules of Emergy Algebra are faithfully accounted for by the mathematical formulation of the Maximum Em-Power Principle.

Under transient conditions, however, the adoption of the Traditional Differential Calculus (TDC) tends to "filter", in *a more or less* marked measure, the "Emerging Properties" of the System described by the concept of Emergy.

In fact, the correlative *Non-Conservative* Algebra manifests that "Emergy" is an "*Emerging Property*" of Self-Organizing Systems, which cannot be expressed in terms of the traditional mental categories, typical of the Traditional Differential Calculus (TDC): *efficient causality*, *logical necessity*, *functional relationships*. Consequently, it cannot be described in terms of the *traditional derivatives* that, at the level of formal language, represent the perfect *reflex* of such "a priori" mental categories.

This evidently suggests the adoption of a different *formal language*, so that the description of Self-Organizing Systems, by means of the Mathematical Formulation of the Maximum Em-Power Principle, could possibly result as being faithfully conform to their "Emerging Quality" (Giannantoni C., 2004a,b).

This is why *a new concept of derivative* was introduced, that is the "*Incipient Derivative*", widely illustrated in (Giannantoni 2001a,b, 2002, 2004a,b).

Its definition, in fact, clearly shows that such a "derivative" is not understood as an "operator", like the derivative (d / dt) in TDC, but it could be better termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being born*.

The first fundamental attempt at adopting such a new form of Derivative was obviously focused on the possible reformulation of the Rules of Emergy Algebra in differential terms. In particular, in terms of the "Incipient" Derivative, when characterized by an appropriate Ordinality.

In this case, the Rules of Emergy Algebra can be reformulated in *Transient Conditions* as follows, and written with reference to the exponential function $e^{\alpha(t)}$ because any function $f(t)$ can always be structured in such a form:

An *Inter-action* can be represented by a "duet" function:
$$(\frac{\tilde{d}}{\tilde{d}t})^2 e^{\alpha(t)} = \left[\overset{\circ}{\alpha}(t), \overset{\circ}{\alpha}(t) \right] \cdot e^{\alpha(t)} \tag{3}$$

A *Co-production* can be represented by a "binary" function:
$$(\frac{\tilde{d}}{\tilde{d}t})^{\frac{1}{2}} e^{\alpha(t)} = \begin{pmatrix} +\overset{\circ}{\alpha}(t) \\ -\overset{\circ}{\alpha}(t) \end{pmatrix} \cdot e^{\alpha(t)} \tag{4}$$

and a *Feed-back* can be represented by a "duet-binary" function:
$$(\frac{\tilde{d}}{\tilde{d}t})^{\frac{2}{2}} e^{\alpha(t)} = \left[\begin{pmatrix} +\overset{\circ}{\alpha}(t) \\ -\overset{\circ}{\alpha}(t) \end{pmatrix}, \begin{pmatrix} +\overset{\circ}{\alpha}(t) \\ -\overset{\circ}{\alpha}(t) \end{pmatrix} \right] \cdot e^{\alpha(t)} \tag{5}$$

where $\overset{\circ}{\alpha}(t)$ represents the incipient derivative of $\alpha(t)$.

It is then easy to recognize that, while the right hand sides of Eqs. (3), (4), (5) represent the *Ordinal Structure* of an Inter-action, Co-production and Feed-back Processes, respectively, the corresponding left hand sides have an *identical formal structure*, always in the form $(\tilde{d}/\tilde{d}t)^q$, where q is the corresponding Ordinality of the *Generative Process*, represented by a rational number which assumes the values of 2, 1/2 and 2/2, respectively. In such a way the Rules of *non-conservative* Emergy Algebra are represented by the "*non-conservative corresponding cardinalities*" associated to each Ordinal Relationship of the different output Processes.

This suggested that the Incipient Derivative $\tilde{d}/\tilde{d}t$ could represent an appropriate mathematical concept in order to express the Generative Activity of any Self-organizing System.

In fact, the Rules of Emergy Algebra, reformulated in terms of the "Incipient" Derivative, could represent a valid basis for a possible generalization of the description of *any System*, made up of an arbitrary number of components.

5. MATHEMATICAL FORMULATION OF THE MAXIMUM ORDINALITY PRINCIPLE

The Rules of Emegy Algebra refer to different possible interactions between *two sub-Systems* and, as previously shown, they can all be represented in the general form

$$(\tilde{d}/\tilde{d}t)_s^{\tilde{k}/\tilde{l}}\{\tilde{r}\} \qquad (6),$$

where $\tilde{k}/\tilde{l}$ stands for $\tilde{2}/1, 1/\tilde{2}, \tilde{2}/\tilde{2}$, while $\{\tilde{r}\}$ is the Relational Space of the System, which in Eqs. (3), (4), (5) is represented by $e^{\alpha(t)}$. This then suggests that is also possible to generalize such a description to a more articulated System. For example, it is possible to write

$$(\tilde{d}/\tilde{d}t)_s^{(\tilde{1}/\tilde{1}1)}\{e^{\tilde{\alpha}(t)}\}^{[\rightarrow}=\{\tilde{0}\} \qquad (7),$$

which, as we will immediately see, can represent the Evolution of the Solar System.

In fact, the number 11 refers to the number of bodies in the Solar Systems, that is: *Sun + 9 Planets + asteroid belt*, while the symbol $^{[\rightarrow}=\{\tilde{0}\}$ indicates that the Solar System, during its time evolution, is always adherent to its initial and habitat conditions.

Eq. (7) can then represent the Mathematical Formulation of the Maximum Ordinality Principle with specific reference to the Solar System. In fact, when formulated in this way, it allowed us to solve several problems that, up to now, are still considered as being unsolvable in Classical Mechanics. For example:

a) The distribution of the Planets in the Solar System

Such distribution, in fact, is still only approximately described by the semi-empirical Bode's Law, which, however, has never found a physical foundation, neither in Classical Mechanics nor in General Relativity.

Vice versa, if the Solar System is modeled as a Self-Organizing System according to Eq. (7), the distribution of the Planets can easily be obtained as the Explicit Solution to the same Eq. (7). In particular, on the basis on an "Emerging Quality Simulator" (EQS), which translates in operative terms the Maximum Ordinality Principle.

The corresponding results are represented in Tab. 2, where it is possible to recognize the improvement of the description when the Solar System is more appropriately considered in relation with its proper Habitat. That is, when it is understood as a part of our Galaxy (Giannantoni C., 2017, cap. 3)

Tab. 2 - Distribution of the Planets in the Solar System

Planets	Bode's Law	Astronomic Data	"Isolated" System	System + Habitat
Mercury	0,4 AU	**0,39 AU**	0.39	**0.39**
Venus	0,7 AU	**0,72 AU**	0.6	**0.70**
Earth	1,0 AU	**1,00 AU**	0.9	**0.98**
Mars	1,6 AU	**1,52 AU**	1.4	**1.50**
Ceres	2,8 AU	**2,77 AU**	2.5	**2.74**
Jupiter	5,2 AU	**5,20 AU**	4.7	**5.05**
Saturn	10,0 AU	**9,54 AU**	8.0	**9.50**
Uranus	19,6 AU	**19,2 AU**	16.0	**19.0**
Neptune **Pluto**	38,8 AU	**30,1 AU** **39,5 AU**	24.0 34.0	**28.9** **38.0**

b) The azimuthal angular distribution of the planetary orbital planes with respect to the Ecliptic

This phenomenon has never found a satisfactory explanation, neither in Classical Mechanics nor in General Relativity. The main reason fundamentally depends on the fact that, in the absence of any explicit solution to the "Three-body Problem", it is impossible to evaluate the exact influence between the reciprocal orbits of the Planets.

The various angles of the orbital planes, in fact, are distributed in a cone of a rather large width (20°), which reduces to 10° only if the extreme Planets (Neptune and Pluto) are "excluded" (because the latter are usually considered as being rather "anomalous"). (ib.)

c) Precessions of the Planets

The Maximum Ordinality Principle is also able to describe another "Irreducible Excess" concerning the Solar System: *The Precessions of the Planets*. (ib.).

General Relativity, in fact, which has given a preliminary answer to this phenomenology, assumes that their values are always "constant" in time and they can be evaluated in terms of a *direct interaction* between *two sole* celestial bodies, such as in the case of Sun and Mercury.

The M.O.P, on the contrary, shows that the Precession of any Planet has to be considered in the context of the Solar System *as a Whole*, when modeled as *one sole* "Self-Organizing System" (ib.).

6. ADOPTION OF THE M.O.P. IN OTHER FIELDS OF ANALYSIS

The M.O.P., whose general formulation is given at paragraph 6.1 and 6.2, do not restrict its valid applicability to the sole Solar System. In fact, it is able to give an elegant solution to several other problems concerning both *non-Living* Systems, Living Systems, and "Thinking" Systems (Human Systems), as synthetically indicated in Tab. 3:

Table 3 - Other Fields of Analysis

Non-Living Systems	Living Systems	Conscious Systems (Human Systems)
i) The "*Three-Body Problem*" (Giannantoni C., 2016) ii) *The angular velocities of the Stars in a Galaxy* (ib.)	i) *Protein Folding* (Giannantoni C., 2010b) ii) *Protein-Protein Interaction* (Giannantoni C., 2015) iii) *Unique Method for Skipping* any *Exon in Duchenne Muscular Dystrophy* (DMD) (Giannantoni, Ordinality.it https://www.semanticscholar.org/paper).	i) *The Three-good two-factor problem in Economics* (Giannantoni C., 2019); ii) *The research for equilibrium conditions in a free-market economy* (ib.); iii) *The Stability of Smart Grids* (ib. and also Giannantoni C., 2012); iv) *Inter-Actions between Man and the Environment* (for example, *The "unexplained" sea level rise over the period 1900-2000* (Giannantoni C., 2019).

6.1. The First Fundamental Equation of the Maximum Ordinality Principle

On the basis of the previous concept of "incipient" derivative, and the Example of the Solar System, the First Fundamental Equation is formulated as follows

$$(\tilde{d}/\tilde{d}t)_s^{(\tilde{m}/\tilde{n})}\{\tilde{r}\}^{[\rightarrow}=\{\tilde{0}\} \qquad (8)$$

$$(\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\} \qquad (8.1)$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration, while $(\tilde{m}/\tilde{n})$ represents its corresponding Ordinality, characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, which reaches its *maximum* when it equals $\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\}$ (as indicated in Eq. (8.1)).

In this respect, it is worth noting that:

i) The *underlined* symbol $(\underline{\tilde{d}/\tilde{d}t})_s$ explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*. This is because it is precisely that which gives origin to its Self-Organization as a Whole;

ii) The symbol "$\overset{[\rightarrow}{=}\{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (8);
- while the symbol "$\overset{[\rightarrow}{=}$", as already anticipated, indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

6.2 The Second Fundamental Equation of the Maximum Ordinality Principle

It is formulated as follows

$$(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}®(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\}\overset{[\rightarrow}{=}\{\tilde{0}\} \qquad (9)$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System. Equation (9), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® with *its proper Generative Capacity* $(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive* Generative Capacity, which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being its proper habitat.

The Maximum Ordinality Principle, formulated in its two fundamental equations, always presents an *explicit solution*, which can also be structured in a more *operative form* by means of an "*Emerging Quality Simulator*" (EQS), already adopted in the case of the Solar System and all the other Systems previously recalled.

7. CONCLUSIONS

The previous exposition enables us to point out *two specific aspects* concerning the Com-possible adoption of the Maximum Em-Power Principle and the Maximum Ordinality Principle. More precisely:

a) The Maximum Em-Power Principle is almost exclusively adopted to describe Self-Organizing Systems in terms of *Energy*, *Economy*, *Environment*, *Natural Resources*, Sustainability, etc.,

b) and, in particular, in *steady state condition*.

As far as the first aspect is concerned, it may be considered as being the *direct reflex* of an "*Elective Choice*", specifically referable to Research Centers, University Faculties and Public Institutions that usually deal with the themes mentioned at point a), especially because in such fields the Maximum Em-Power Principle is recognized as being particularly appropriate to suggest strategic decisions.

The second aspect, however, can be seen as a *form of fidelity* to the original version of The Maximum Em-Power Principle as proposed by H.T. Odum, however, and in particular, only in *steady state condition*.

The two abovementioned aspects will be now considered in the light of the fact that the Maximum Ordinality Principle has a much wider and general validity, in any field of analysis, both in *steady state and transient conditions*.

In this respect it is then fundamental to point out that, on the basis of the respective properties of the Maximum Em-Power Principle and the Maximum Ordinality Principle, it is possible to assert that, strictly speaking, from a *logical point of view* they can be considered as being *Com-possible*. That is, without any reciprocal "exclusion".

The Maximum Ordinality Principle, in fact, is based on *Generative Logic*, which has not the property of "the perfect induction". The same happens for the Maximum Em-Power Principle, which is equally based on *Generative Logic*.

However, apart from their "Com-possibility" from a *logical point of view*, another import aspect is whether they lead to the same results or not. That is whether they are also "Com-possible" from *an operative point of view*.

In this respect it is particularly important to distinguish *two different operative conditions*:

a) When the two Principles are adopted to describe any System *under steady state conditions*, they can be considered as being substantially equivalent from *an operative point of view*.

In fact, as previously said, the Rules of *non-conservative* Emergy Algebra, which are specific of the Maximum Em-Power Principle, have their appropriate correspondence with the "*non-conservative*" *cardinal relationships*, which, in *operative terms*, can always be associated to each Ordinal Relationship specific of the Maximum Ordinality Principle.

This means that, *under steady state conditions*, it is possible to adopt the one or the other in substantially equivalent operative terms.

b) Under *transient conditions*, however, the adoption of the Traditional Differential Calculus (TDC) in the context of the Maximum Em-Power Principle, *generally* tends to "filter", in *a more or less* marked measure, the "Emerging Properties" of the System, although its description is fundamentally based on the concept of Emergy.

Such a general trend, in fact, presents an *exception* only in very special circumstances. For example, when the Emergy of the System, in transient conditions, is represented by an exponential function $e^{\alpha(t)}$, where the exponent $\alpha(t)$ is linear. This in fact is the *only case* in which the *traditional derivatives* formally coincide with the *incipient derivatives*.

Consequently, even if in this case the results obtained from the two distinct approaches *may occasionally coincide* from a *quantitative point of view*, the results obtained on the basis of the M.Em-P.P are always a *necessary consequence* of a *logical and physical* process. Whereas, those obtained from the M.O.P., which is based on the incipient derivative, are understood as the *exit* of a *Logical Generative Process*, whose specific Ordinalities are operatively represented in terms of their "*associated cardinalities*", and thus only *occasionally* they may lead to the same trend.

In synthesis: the Maximum Ordinality Principle and the Maximum Em-Power Principle are always *Com-possible* from a *logical point of view*. However, in spite of their logical *Com-possibility*, they do *not always* present a corresponding *Com-possibility from an operative point of view*.

In fact, the basic difference between the Maximum Em-Power Principle and the Maximum Ordinality Principle manifests in their different respective capacity of describing the "Emerging Quality" of Self-Organizing Systems *under transient conditions*, when the latter differ from the specific case previously indicated.

This means that, under transient conditions, it is advisable to start the analysis on the basis of the Maximum Ordinality Principle, because it has *no limitations in principle* and, after that, eventually to repeat the analysis in terms of the Maximum Em-Power Principle, in order to point out their possible correlative differences of results.

In all cases both the Maximum Ordinality Principle and the Maximum Em-Power Principle always keep, *in principle*, their capacity of describing the "Emerging Quality" of Self-Organizing Systems. That is, precisely that "Emerging Quality" which is not "recognized" by the traditional Principles of Thermodynamics. Even if in the case of the M.Em-P.P., and in some particular *transient conditions*, such a capacity may result as being *partially reduced*.

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