The Maximum Ordinality Principle, from the "Incipient" Derivative to EQS Simulator

Abstract

This document represents a synthesis and, at the same time, a further development, finalized to a "relaunch", of the Maximum Ordinality Principle, as previously illustrated in the various papers (from 1999 to 2020) presented at the Biennial Emergy Conferences (University of Florida).

The Maximum Ordinality Principle (Giannantoni, 2010a), in fact, is nothing but the reformulation of the Maximum Em-Power Principle (Odum, 1994a,b,c), given however in a more general form by means of a new concept of derivative, the "incipient" derivative, whose mathematical definition has already been presented in (Giannantoni, 2001a, 2002, 2004, 2008, 2009b, 2010a).

In this way both Emergy and Transformity are replaced by the concept of Ordinality. This is the reason why the principle was renamed as the Maximum Ordinality Principle (Giannantoni, 2010a,b).

Consequently, on the basis of the Mathematical Formulation of the Maximum Ordinality Principle (Giannantoni, 2010a) and, in particular, its adoption as "One Sole Reference Principle" (Giannantoni, 2014), we can now present, in more details, the radically New Perspective that such a Principle offers to Modern Science. That is: "*Every System is a Self-Organizing System*".

In order to give a clear presentation of the fundamental differences between such a New Perspective with respect to the Traditional Scientific Approach, the document will start from the consideration of a synoptic picture of the basic characteristics of the two mentioned Scientific Approaches (see Tab. 1), successively analyzed and compared, in more detail, in the context of the document.

1. Fundamental Characteristics of the Two Scientific Approaches

In this respect, it is worth starting from recalling that Self-Organizing Systems and their "emerging properties" began to be studied by L. Boltzmann toward the end of XIX century. Several other Authors (e.g. A. Lotka) dealt with such a theme. However, Self-Organizing Systems received the most significant contribution by H.T. Odum (from 1955 on), with the genial introduction of a more appropriate formal language.

The consequential faithful developments of Odum's approach have led us to the formulation of a unique general Principle, the Maximum Ordinality Principle (M.O.P.), which is able to describe, by itself, the behavior of any given System as a Self-Organizing System: both "*non-living*" Systems, "*living*" Systems and "*thinking*" Systems too (e.g. Human Systems).

Such a conclusion then results as being deeply different from that of Modern Science, which, from Newton on, is persistently orientated at describing any known system as it were a "mechanism".

The present document, after having synthetically recalled the formulation of the M.O.P. and after having pointed out its corresponding descriptive advantages, will focus on the intrinsic new perspective offered by the M.O.P. especially *in thinking, decision making and acting*, with respect to the Traditional Approach. In particular, with reference to any form of relationship between Man and his surrounding environment.

In particular, and with reference to this fundamental aspect, the basic differences between the two afore-mentioned perspectives will be brought out by comparing, on the one hand, "*side effects*" (related to the Traditional Approach) and, on the other hand, the "*Emerging Exits*" (specifically pertaining to the New Approach).

Let us thus consider first the Traditional Approach that characterizes Modern Science.

1.1 The Traditional Scientific Approach

Modern Science is characterized by a persistent and progressively ascendancy toward ever more general Physical Laws and Principles.

However, before any formulation of a single hypothesis or a physical theory, Modern Science (let us say, from Newton on) adopts three fundamental *pre-suppositions* (see Tab. 1): the *causality principle* (also termed as "efficient causality"), *classical logic* (also termed as "necessary logic"), and *functional relationships* (between the various parts of any System analyzed).

On the basis of such fundamental presuppositions, and only after having developed a strictly conform consequential *formal language* (that is the Traditional Differential Calculus (TDC)), Modern Science progressively ascends toward ever more general Physical Laws and Principles:

i) from Phenomenological Laws (e.g. Kepler's Laws); ii) to Physical Laws specific of each Discipline (e.g. Newton's Laws, Maxwell's Equations, etc.); iii) up to the three well-known Thermodynamic Principles.

Such a progressive development has given origin to a hierarchy of a multiplicity of *quantitative* Physical Laws and Principles, in particular as a consequence of the first basic presupposition: the *causality principle*. This Principle, in fact, has led Modern Science to introduce "different causes" in different Disciplines. The Principle of causality, in fact, tends to "sub-divide" the entire phenomenology (at present known) in different "branches", precisely because, on the basis of such a presupposition, it leads Scientists to research for the most "appropriate causes" pertaining each specific set of phenomena each time considered.

In this way, Modern Science persistently propends to show that: "*Every System is a mechanism*".

Such a conclusion, however, although confirmed by experimental results, can be considered as being valid *only* from an *operative* point of view, but not from an *absolute point of view*. This is because "necessary logic" (adopted as second basic presupposition) does not admit any form of "*perfect induction*" (see Popper's *Falsification Principle***)**.

In fact, as synthetically illustrated in Tab. 1, in the strict contest of "necessary logic":

i) after having formulated a single or more hypotheses (such as in the case of a Theory);

ii) after having formalized them in an appropriate formal language (faithfully conform to the three above-mentioned basic presuppositions);

iii) after having drawn the consequential conclusions

iv) and after having also obtained experimental confirmations of the previous formal conclusions;

v) it is impossible, *in any case whatsoever*, to assert the *uniqueness* of the *inverse* process. That is: it is impossible to show that the hypotheses adopted are the *sole* and *unique* hypotheses capable to explain those experimental results.

This is precisely because of the *absence*, in "*necessary*" logic, of any form of *perfect induction*.

In fact, only in the presence of a *perfect induction* it would be possible to assure the *uniqueness* of the *inverse* process and, thus, to transform the adopted hypotheses into an *absolute* perspective.

This means that Modern Science, precisely because based on *necessary logic*, should always be "open" to recognize that *there always exist* many other *possible* Approaches (in principle *infinitive*) capable to interpret the same experimental results.

At this stage, after having synthetically recalled the basic characteristics of Modern Science, we can analyze in more detail the fundamental properties of the New Perspective, synthetically indicated in parallel (for a better comparison) in the right hand side of Tab. 1.

Basic Presuppositions	***"Emerging Quality"* of Self-Organizing Systems**
1) causality principle (efficient causality) 2) classical logic (necessary logic) 3) functional relationships	1') Generative Causality 2') Adherent Logic (Emerging Conclusions) 3') Ordinal Relationships

	Development of an appropriate Language
d/dt is the corresponding formal translation $f(t)$ represents a *functional relationship*	- L. Boltzmann, A. Lotka - H. T. Odum: Emergy Algebra and M. Em-P. P. - Further developments in transient conditions - Introduction of the "Incipient" derivative $\tilde{d}/\tilde{d}t$

- Thermodynamic Principles (1st , 2nd, 3rd) **- Physical Laws (specific for each Discipline)**	**The Maximum Ordinality Principle**
Every System is a "Mechanism" **Hypotheses** ↓ **Mathematical Formalization** ↓ **Conclusions** ↓ **Confirmation by experimental results**	**- is applicable to *any* Field of analysis: *non-living* Systems, *living* Systems, "*thinking*" Systems (e.g. Human Systems)** **- at *any space-time scale* and in *variable conditions*** **- it also offers a *more appropriate* description of any given System and its surrounding habitat** **Every System is a "Self-Organizing System"**

Tab. 1 - Synoptic comparison between the basic presuppositions of the two *differential formal languages* and their main corresponding fundamental characteristics

1.2 "Emerging Quality" of Self-Organizing Systems and Adoption of New Mental Categories

After having synthetically recalled the basic characteristics of Modern Science and its corresponding formal language, we can now analyze the fundamental properties of a New Scientific Perspective, which leads to the introduction of a new Formal Language, the *Incipient Differential Calculus* (IDC). As anticipated, the fundamental properties we are referring to are synthetically indicated in parallel (for a better comparison) in the right hand side of Tab. 1.

Such a New Scientific Perspective is based on the *phenomenological* "Emerging Quality" of Self-Organizing Systems (Giannantoni, 2016). This represents the fundamental aspect that leads to the adoption of the corresponding *new mental categories* (shown in Tab. 1).

The expression "*Emerging Quality of Self-Organizing Systems*" refers to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So that they usually say: "*The Whole is much more than its parts*".

Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*.

This evidently suggests a *radically new* gnosiological perspective, which corresponds to recognize that: "*There are processes, in Nature, which cannot be considered as being pure* "*mechanisms*".

This also leads, *in adherence*, to the adoption of "*new mental categories*"[1] and, correspondently, to the development of a completely *new formal language*, so that the description of Self-Organizing Systems might result as being faithfully conform to their "Emerging Quality"

[1] These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are chosen only "a posteriori", on the basis of the "Emerging Quality" previously recognized. "*Generative Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Adherent Logic*", correspondently, refers to the capacity of our mind to draw "*emerging conclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to

2. The Progressive Development of an Appropriate Formal Language

L. Boltzmann was the first who attempted at describing Self-Organizing Systems in more appropriate formal terms, by proposing the adoption of a new Thermodynamic Principle: The Principle of Maximum Exergy *Inflow* to the System (Boltzmann 1886).

Some years later, A. Lotka (1922-1945) reformulated such a Principle in the form of: The Principle of Maximum Exergy *Flow through* the System (Lotka, 1922a,b, 1945).

Both such attempts were not perfectly successful, because still based on the concept of Exergy, which is a quantity that is strictly pertaining to Classical Thermodynamics. Consequently, it re-proposes, by itself, the concepts of *efficient causality*, *logical necessity*, *functional relationships*.

A really *new formal language* only appears with H. T. Odum, with the genial introduction of Emergy (*Em*), defined as Exergy (*Ex*) by Transformity (*Tr*)

$$Em = Ex \cdot Tr \qquad (2.1).$$

Equation (2.1) clearly shows that Emergy is *still* based on "Exergy". However:

i) *Quality Factor Tr* "*Transforms*" *Ex* into a *new physical quantity*: <u>Emergy;</u>

ii) The latter in fact is not defined in "functional terms", but only by "*assignation Rules*" (Brown and Herendeen, 1996);

iii) This is precisely because *Tr* is expressed by means of a *non-conservative Algebra*;

iv) Thus the output "excess" of the three Fundamental Process in Emergy Analysis (Co-Production, Inter-Action, Feed-Back) is always understood as being "irreducible" to its specific inputs in *mere functional terms*.

This means that <u>Emergy</u> is able to represent the "Emerging Quality" of Self-Organizing *Processes.* Consequently, the general enunciation of the *Maximum Em-Power Principle* (Odum, 1994a,b,c) can *equally be referred,* at a phenomenological level, to the *corresponding maximization tendency* of the "Emerging Quality" on behalf of *Self-Organizing Systems*.

The Maximum Em-Power Principle, however, had not a corresponding and specific formulation under *variable conditions*. On the other hand, such a formulation in *variable conditions* could not be given in terms of the Traditional Differential Calculus, because the traditional derivatives, as a consequence of their conceptual basic presuppositions (see Tab. 1), are not properly apt at representing the "generative" behavior of "Self-Organizing Systems", and consequently they tend to partially "filter" such a "generative" behavior.

This is why, in order to achieve an appropriate mathematical formulation of the Maximum Em-Power Principle, I introduced the concept of "*Incipient Derivative*", defined as (Giannantoni, 2001a,b, 2002, 2004a,b)

$$\left(\frac{\tilde{d}}{\tilde{d}\,t}\right)^{\tilde{q}} f(t) = \underset{\tilde{\Delta} t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta} t}\right)^{\tilde{q}} \circ f(t) \qquad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \qquad (2.2).$$

a definition that will be illustrated in detail in the next paragraph.

However, it is already possible to anticipate that such a definition shows that the "*Incipient Derivative*" is not an "operator", like the traditional derivative (d/dt), but it could be termed as a "*generator*", because it describes a Process in its same act of being born (Giannantoni, 2006, 2008a, 2010a).

their logical premises, although persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, which will be illustrated in more details later on, with reference to any *Generative Process*.

The Mathematical Formulation of the M. Em-P. Principle in terms of *Incipient Derivatives* was preliminarily given in (Giannantoni, 2001b), and afterwards, in a more articulated form, in a specific book co-financed by the Center for Environmental Policy (Giannantoni, 2002).

During the successive eight years (2002-2010), such a mathematical formulation was adopted in several Disciplines, such as *Classical Mechanics*, *Quantum Mechanics*, *General Relativity*, *Chemistry*, *Biology*, *Economics* and the corresponding results were reunited *in two books* (titled: *"Lightness of Quality"* (Giannantoni, 2007) *and "Ascendency of Quality*" (Giannantoni, 2008b).

At the end of this wide range of applications, I realized that it was possible to give a more general formulation of the Maximum Em-Power Principle, in the form of the "*Maximum Ordinality Principle*" (Giannantoni, 2010a).

For the sake of clearness, the Rational of such a generalization process, articulated in a few logical steps, is recalled in the next sections.

3. The Incipient Derivative of Ordinality $\tilde{q}$

The "Incipient" Derivative of a given Ordinality $\tilde{q}$, whose definition previously introduced is here recalled for the sake of clarity

$$(\frac{\tilde{d}}{\tilde{d}t})^{\tilde{q}} f(t) = \underset{\tilde{\Delta}t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \qquad \text{for } \tilde{q} = \tilde{m}/\tilde{n} \qquad (3.1)$$

will be illustrated by considering first its *general properties* and, immediately after, its more *specific properties*.

To this purpose it is worth preliminary pointing out that the concept of "Ordinality" refers to two "distinct" concepts, which however are considered as being *one sole entity*, that is as a *Whole*. These are: its "*cardinality*" and its "*ordinal genetic relationships*". This means that the Ordinality $\tilde{q}$, synthetically represented as $\tilde{q} = \tilde{m}/\tilde{n}$ (as in Eqs. (2.2) and (3.1)), in reality it has to be more properly understood as

$$\{\tilde{m}/\tilde{n}\} = \{k, (\tilde{m}/\tilde{n})\} \qquad (3.2)$$

in which:

- k represents its *cardinality*
- while $(\tilde{m}/\tilde{n})$ represents its *Ordinal Genetic Relationships*, where the *round brackets* expressly indicate that they represent *only a part* of the concept of Ordinality, understood as a Whole. In fact, the first member of Eq. (3.2) is represented in *curly brackets*, precisely because this symbol is usually adopted to indicate the concept of a Whole.

The Ordinal Genetic Relationships $(\tilde{m}/\tilde{n})$ can also more synthetically termed as "Ordinal Relationship", not only because they are not "functional" Relationships, but especially because the adjective "Ordinal" also indicates that they are precisely those Relationships that give the most significant contribution to the definition of the general concept of Ordinality understood as a Whole.

3.1 General Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Definition (2.2) clearly shows what we have synthetically anticipated, that is: the "*Incipient Derivative*" is not an "operator", like the derivative (d/dt) in the Traditional Differential Calculus (TDC), but it could be termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being born* (Giannantoni, 2001a,b, 2002, 2004a,b, 2006, 2008a, 2010a). In fact:

i) The sequence of the symbols is now interpreted according to the *direct priority* of the three elements that constitute its definition (*from left to right*). This is the reason why they acquire a completely new different meaning with respect to the traditional one;

ii) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;

iii) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "*threshold*" of that "*ideal window*" from which we observe and describe the considered phenomenon;

iv) The symbol $\tilde{\Delta} t : 0 \to 0^+$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) in its proper act of being born, as a Generative Process;

v) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). Thus the ratio which appears in Eq. (3.1) indicates not only a quantitative variation in time, but both the variation in Quality and quantity;

vi) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *Exit* of such a process will keep "memory" of its genetic origin. This is because, besides its quantity, it will result as being Ordinally structured (as shown at the next paragraph 3.2.2) according the indication of such an exponent. The latter in fact precisely expresses how each part of the output is *genetically Ordered* to the Whole and, at the same time, *how each part is related to all the others* in terms of *Ordinal Harmony Relationships* (illustrated at paragraph 5.6);

vii) In this way the "incipient" derivative represents the *Generativity of the considered Process*, that is the output "excess" (per unit time) characterized by both its *Ordinal Genetic Relationships* and its related *cardinality*, while the sequence of the symbols in its definition can be interpreted as representing a *unique inter-action process* between the same;

viii) The above-mentioned reasons clearly show why the "Incipient" Derivative, precisely because of such properties, is able to *unify* (and, at the same time, to specify) the description of the various Self-Organizing Processes of the surrounding World, when they are explicitly understood in terms of Quality.

3.2 Specific Properties of the "Incipient" Derivative of Ordinality $\tilde{q}$

Let us start from considering first its *specific cardinality k*.

3.2.1 The "Incipient" Derivative of cardinality *k*

On the basis of Definition (3.1), the exit of the incipient derivative of Ordinality *k* is (Giannantoni, 2004b)

$$\frac{\tilde{d}^k}{\tilde{d}\,t^k} f(t) = \left(f^{'}(t) \tilde{/}\, f(t)\right)^k \cdot f(t) \qquad (3.2.1).$$

In fact, through successive formal passages, we have that

$$(\tilde{\delta}-1)\, f(t) = f\left(t + \tilde{\Delta} t\right) - f(t) = f(t) + f^{'}(t) \cdot \tilde{\Delta} t - f(t) = f^{'}(t) \cdot \tilde{\Delta} t \qquad (3.2.2)$$

and, consequently

$$(\tilde{\delta}-1)/\tilde{\Delta}t=\{f^{'}(t)/f(t)\} \qquad (3.2.3).$$

Such an expression, when introduced in the Definition (3.1), gives

$$\lim_{\tilde{\Delta}t:0\to 0^{+}}\cdot\left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{k}}\cdot f(t)=\left(f^{'}(t)/f(t)\right)^{\tilde{k}}\cdot f(t) \qquad (3.2.4).$$

Such an explicit formal process shows that the definition of the "*Incipient*" Derivative of cardinality *k* is based on a concept of limit, which however is "*prior*" with respect to the considered function**,** and only *after* its corresponding evaluation it is specifically referred to the considered function.

It is also worth adding that in Eqs. (3.2.2) and (3.2.3) we have adopted the simple notation $f^{'}(t)$, which in reality is more typical of TDC. It is thus now important to point out, apart from its similarity, what are its specific differences with respect to the "Incipient" Derivative $\tilde{f^{'}}(t)$.

If we consider in fact the "Incipient" Derivative of cardinality *k* of the exponential function, that is, if we assume that $f(t)=e^{\alpha(t)}$, on the basis of Eq. (3.2.4) we get

$$(\frac{\tilde{d}}{\tilde{d}t})^{k}e^{\alpha(t)}\overset{*}{=}e^{\alpha(t)}\cdot[\overset{\circ}{\alpha}(t)]^{k} \qquad (3.2.5)$$

in which the specific symbology adopted $\overset{\circ}{\alpha}(t)$ is finalized to point out that, even if on the basis of Eq. (3.2.4) the first order "Incipient" Derivative (now indicated with $\overset{\circ}{\alpha}(t)$) coincides with the traditional derivative $\alpha'(t)$, the *logical processes* that lead to such identical (quantitative) results are radically different. A difference which, in particular, is also pointed out by the adoption of the symbol $\overset{*}{=}$, which reminds us that any "Incipient" Derivative is always the *exit* of a *Generative Logical Process* and not of a *necessary* logical process.

Eq. (3.2.5) can thus preferentially adopted as *general definition* of the "Incipient" Derivative of cardinality *k*. This is because any function $f(t)$ can always be written in the form $f(t)=e^{\ln f(t)}=e^{\alpha(t)}$.

Such a formal representation, in fact, leads to the same result as that of Eq. (3.2.4). However, such a formal representation of the "Incipient" Derivative of cardinality *k* is much more "Ostensive", as we will see, when we will introduce the general definition of Relational Space and, even more, when we will deal with the explicit solution to the Maximum Ordinality Principle.

At the same time, it is also particularly apt at showing the deep differences between the cardinal values of the "Incipient" Derivatives and those pertaining to the traditional derivatives.

In fact, if we compare the traditional derivative of order *n* of the function $e^{\alpha(t)}$, evaluated according to Faà di Bruno's formula

$$(\frac{d}{dt})^{n}e^{\alpha(t)}=e^{\alpha(t)}\sum\frac{n!}{k_1!k_2!\ldots k_n!}\cdot\left(\frac{\dot{\alpha}}{1!}\right)^{k_1}\left(\frac{\ddot{\alpha}}{2!}\right)^{k_2}\cdot\cdot\left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \qquad (3.2.6)$$

with the "Incipient" Derivative of the corresponding cardinality *n*

$$(\frac{\tilde{d}}{\tilde{d}t})^n e^{\alpha(t)} \stackrel{*}{=} e^{\alpha(t)} \cdot [\overset{\circ}{\alpha}(t)]^n \qquad (3.2.7),$$

we can easily recognize that they are *deeply different*. And, even if in some cases the two derivatives of the same order *k* coincide (for instance when $\alpha(t)$ is linear), such a coincidence has always to be seen in the light of the symbol $\stackrel{*}{=}$ in Eq. (3.2.7), which reminds us that any "Incipient" Derivative is always the *exit* of a *Generative Logical Process* and not of a *necessary* logical process. A concept that is contextually and specifically underlined in Eq. (3.2.7) by the explicit adoption of the "notation" $[\overset{\circ}{\alpha}(t)]^n$.

3.2.2 The Ordinal Genetic Relationships $(\tilde{m}/\tilde{n})$ of the "Incipient" Derivative of Ordinality $\tilde{q}$

As already anticipated, beside its proper cardinality *k*, the "Incipient" Derivative of Ordinality $\tilde{q}$, according to Eq. (3.1), is characterized by the genesis of its corresponding *Ordinal Genetic Relationships*, whose specific indication is represented by $(\tilde{m}/\tilde{n})$.

In this respect, it is worth pointing out that the symbol $f(t)$ does not represent anymore a simple "function", such as in the case of TDC, but it represents a *Physical Entity*, of *Generative Nature*. Consequently a more appropriate symbol should be $\tilde{f}(t)$, where the "tilde" notation specifically reminds us its *Generative Nature*.

More specifically, in the general context of Self-Organized Systems, the symbol $\tilde{f}(t)$ will be more properly understood as being representing the *Relational Space* of a given System, as it will be shown in the next paragraphs.

After these due premises, we can assert that the "Incipient" Derivative of Ordinality $\{\tilde{q}\} = \{k, (\tilde{m}/\tilde{n})\}$ describes a *Generative Process* which, with reference to a given System, is characterized by both its *cardinal* and "*internal genetic properties*", and it can be represented as follows

$$(\frac{\tilde{d}}{\tilde{d}t})^{\{k,(\tilde{m}/\tilde{n})\}} e^{\alpha(t)} \stackrel{*}{=} e^{\alpha(t)} \cdot \left\{ \begin{pmatrix} [\overset{\circ}{\alpha}_{11}(t)]^k \\ [\overset{\circ}{\alpha}_{21}(t)]^k \\ \cdots\cdots \\ [\overset{\circ}{\alpha}_{m1}(t)]^k \end{pmatrix}, \begin{pmatrix} [\overset{\circ}{\alpha}_{12}(t)]^k \\ [\overset{\circ}{\alpha}_{22}(t)]^k \\ \cdots\cdots \\ [\overset{\circ}{\alpha}_{m2}(t)]^k \end{pmatrix} \begin{pmatrix} \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \end{pmatrix} \begin{pmatrix} [\overset{\circ}{\alpha}_{1n}(t)]^k \\ [\overset{\circ}{\alpha}_{2n}(t)]^k \\ \cdots\cdots \\ [\overset{\circ}{\alpha}_{mn}(t)]^k \end{pmatrix} \right\} \qquad (3.2.8)$$

where:

- k represents the *cardinality* of the "Incipient" Derivative

- $\overset{\circ}{\alpha}_{ij}(t)$ are the *genetic characteristics* of the considered system, which are highlighted by the *Generative Process* described by the "Incipient" Derivative. For this reason they should more properly be represented as being characterized by a "tilde" notation. However, for the sake of a simpler notation, it has been omitted, and thus it is simply understood

- such genetic characteristics $\overset{\circ}{\alpha}_{ij}(t)$ are generally referred to the specific properties of the Relational Space $\alpha(t)$ and are evidently characterized by the initial and boundary conditions of the System

- at the same time the "matrix" which appears in the second member of Eq. (3.2.8) *is not* a traditional matrix. In fact it is an "*Ordinal*" Matrix, whose various elements are related between them through Ordinal Relationships, of Genetic Nature, in the form *N* Co-Generated genetic properties (vertical columns), further related between them in the form of *N* Interaction Ordinal Relationships (parallel sequence of the N column). The "*Ordinal*" Matrix thus represents an *Ordinal Cooperation* of *N* Co-Productions and their associated *N* Inter-actions.

In this way the various elements form *One Sole Entity*, faithfully represented by the abovementioned *Ordinal Matrix*. A concept that is explicitly pointed out, also in this case, by the adoption of *curly brackets*.

In addition, in order to distinguish such an *Ordinal Matrix* from a traditional matrix, from now on, for the sake of brevity, it will be simply termed by means of the single term "*Matrioska*".

The structure of the "Incipient" Derivative (3.2.8) is then able to Ostend even more clearly the concepts previously anticipated. That is:

- the symbol $(\tilde{m}/\tilde{n})$ represents the *Ordinal Genetic Relationships* that characterize the "Incipient" Derivative, where the round brackets expressly indicate that they represent only *a part* of the concept of Ordinality, which vice versa is understood, by itself, as a Whole;
- In fact, for this reason, in Eq. (3.2.8) the latter concept is represented by means of the adoption of *curly brackets*;
- The *Ordinal Genetic Relationships* can also more synthetically termed as "*Ordinal Relationship*", both because they are not, in themselves, "functional" Relationships, but especially because the adjective "Ordinal" clearly indicates that they are precisely those that give the most significant contribution to the definition of the general concept of Ordinality;
- In addition, Eq. (3.2.8) allows us to point out that, when we preliminary introduced the concept of *cardinal* "Incipient Derivative", this was represented as a simple and proper *mathematical concept*, which, in this sense, has some similarities with that of a traditional derivative. This is why it was possible to continue to adopt the term "function" and the correlative symbol $f(t)$, even if it was well clear the profound difference between the correlative Logical Process adopted;

- Vice versa, when we consider the "Incipient" Derivative of Ordinality $\tilde{q}$, its meaning, when considered in the descriptive context of Self-Organizing Systems, is more properly referable as the description of a *Generative Process*;
- Consequently, in such a case it is more appropriate to consider Eq. (3.2.8) as representative of a *Generative Process*, which highlights the Genetic Properties of a *Physical Entity* that, in the case of a Self-Organizing System, it is usually represented by the proper *Relational Space* of the System;
- So that, to take into account the abovementioned different aspects between the two considered Derivatives, in general it is preferable to adopt the synthetic tilde notation $\tilde{f}(t)$, in order to more specifically indicate, in addition, that the considered System is already the Exit of *a previous* Generative Process.

3.2.3 Specific Properties of the "Incipient" Derivative when understood of *Higher Ordinality*

The Ordinality of the "Incipient" Derivative, as previously defined (see Eq. (3.2)), represents the most frequent form of Ordinality of the Self-Organizing Systems usually considered.

However, in particularly cases (especially in "Living" Systems), it may be characterized by a more "articulated" structure. For example, its cardinality can directly be associated to a correlative Ordinality $\tilde{2}/\tilde{2}$, corresponding to an "*additional*" Coproduction-Interaction Process.

In such a case the Ordinality $\tilde{q}$ will be then represented as

$$\tilde{q} = \{k \uparrow \tilde{2}/\tilde{2}; (\tilde{m}/\tilde{n})\} \tag{3.2.9}$$

in order to have, in such a way, a more adherent representation of the Internal Generativity of the System under consideration.

In this respect, however, some examples of more articulated forms of "Incipient" Derivative, with reference to *particularly complex* "Living" System, are illustrated in (Giannantoni, 2018).

4. Mathematical Formulation of the Maximum Ordinality Principle

The Maximum Ordinality Principle (M.O.P.), whose verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", is formulated by means of two fundamental equations, which are so *strictly related to each other,* so as to form *a Whole* (Giannantoni, 2010a, 2012, 2014a,b, 2016, 2017):

4.1 The First Fundamental Equation of the Maximum Ordinality Principle

On the basis of the previous concept of "Incipient" Derivative, the First Fundamental Equation is formulated as follows

$$(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{m}/\tilde{n}\}}\{\tilde{r}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{4.1}$$

$$(\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\} \tag{4.1.1}$$

where $\{\tilde{r}\}$ is the *Relational Space* of the System under consideration (see paragraph 5.1), while $\{\tilde{m}/\tilde{n}\} = \{k, (\tilde{m}/\tilde{n})\}$ represents its corresponding Ordinality, while $(\tilde{m}/\tilde{n})$ indicates the *Ordinal Genetic Relationships* characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, and the Maximum Ordinality is reached when $(\tilde{m}/\tilde{n})$ equals $\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{N}/\tilde{N}\}$ (as indicated in Eq. (4.1.1)).

In this respect, it is worth noting that:

i) The *underlined* symbol $(\underline{\tilde{d}/\tilde{d}\,t})_s$ explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*. This is because it is precisely that which gives origin to its Self-Organization as a Whole;

ii) The symbol "$\overset{[\rightarrow}{=} \{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (4.1);
- while the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its *Generative Evolution*, is persistently "adherent" to its "origin and habitat" conditions.

4.2 The Second Fundamental Equation of the Maximum Ordinality Principle

It is formulated as follows

$$(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\} \circledR (\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\} \overset{[\rightarrow}{=} \{\tilde{0}\} \tag{4.2}$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System. Equation (4.2), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product ® (defined at paragraph 5.1) with *its proper Generative Capacity* $(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive* Generative Capacity, which *at any time*, is always adherent to the origin and habitat conditions of the Second Fundamental Equation.

This is an aspect which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being part of its proper habitat.

The Maximum Ordinality Principle, in its two fundamental equations, *always* presents an *explicit solution*. This will be presented:

a) by preliminarily illustrating its basic elements

b) then by formulating the correlative solution in explicit terms

c) finally, at the end of the document, the general explicit solution to the M.O.P. will also be presented and structured in a corresponding *operative form*, so that it may result as being more directly and easily adopted in analysing any System under consideration.

5. Explicit Solution to the Mathematical Formulation of the Maximum Ordinality Principle

In order to show the explicit solution to the Maximum Ordinality Principle, it is worth recalling the fundamental concepts pertaining to the *Relational Space* of a System.

5.1 The Relational Space of a System

In this respect it is fundamental to recall that the symbol $\{\tilde{r}\}$ in Eq. (4.1) represents the *Relational Space* of the System, which obviously depends on the Nature of the System analyzed.

We can then start from the consideration of a System whose *Relational* Space is characterized, for example, by the following three topological coordinates $\{\tilde{\sigma},\ \tilde{\varphi},\ \tilde{\vartheta}\}$.

Such a hypothesis is surely valid in the case of a "*non-Living*" System. Nonetheless, it is also valid in the case of a "*Living*" System too. Whereas, in the case of "Conscious" Systems, the three coordinate will surely be different.

For example, in the case of the Economic Analysis of European Community, with its 27 States, the variables could be (K, L, N), that is *Kapital*, *Labour* and *Natural Resources*, as shown in (Giannnatoni, 2019).

In all cases whatsoever, the three topological coordinates $\{\tilde{\sigma},\ \tilde{\varphi},\ \tilde{\vartheta}\}$ are always considered as *the exit of a Generative Process* (this is the reason why for the tilde notation), and we always have that

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\tilde{\sigma}®\tilde{i}\oplus\tilde{\varphi}®\tilde{j}\oplus\tilde{\vartheta}®\tilde{k}\}} \qquad (5.1.1).$$

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_s = \{\tilde{\rho}®\tilde{i}®e^{\tilde{\varphi}®\tilde{j}}®e^{\tilde{\vartheta}®\tilde{k}}\} = \{e^{\tilde{\sigma}®\tilde{i}}®e^{\tilde{\varphi}®\tilde{j}}®e^{\tilde{\vartheta}®\tilde{k}}\} = e^{\{\tilde{\sigma}®\tilde{i}\oplus\tilde{\varphi}®\tilde{j}\oplus\tilde{\vartheta}®\tilde{k}\}} = e^{\tilde{\alpha}(t)} \qquad (5.1.2),$$

where the traditional versors $\vec{i}$, $\vec{j}$, $\vec{k}$ are now replaced by three unit *spinors* $\tilde{i},\ \tilde{j},\ \tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i}®\tilde{i}=\oplus 1 \qquad \tilde{i}®\tilde{j}=\tilde{j} \qquad \tilde{i}®\tilde{k}=\tilde{k} \tag{5.1.3}$$

$$\tilde{j}®\tilde{i}=\tilde{j} \qquad \tilde{j}®\tilde{j}=\Theta 1 \qquad \tilde{j}®\tilde{k}=\tilde{k} \tag{5.1.4}$$

$$\tilde{k}®\tilde{i}=\tilde{k} \qquad \tilde{k}®\tilde{j}=\tilde{k} \qquad \tilde{k}®\tilde{k}=\Theta 1 \tag{5.1.5}$$

where the symbols $\oplus$ and ® express more intimate relationships between the same spinors: both in terms of sum $\oplus$ and in terms of (relational) product ® with respect to the case of traditional versors $\vec{i}$, $\vec{j}$, $\vec{k}$.

So that representation (5.1.1) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one real unit $(\tilde{i})$ and two imaginary units ($\tilde{j}$ and $\tilde{k}$).

5.2 The Generative Capacity of the System

As already anticipated, the incipient derivative $(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{m}/\tilde{n}\}}$, when it is *underlined*, explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*.

This is precisely because under these conditions it represents the Self-Organization of the System as a *Whole.*

At the same time this is also the reason why, differently from the traditional "incipient" derivative, in our case the "Incipient" Derivative is directly *referred to the exponent* of the Relational Space, that is

$$e^{(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{m}/\tilde{n}\}}\{\sigma®\tilde{i}\oplus\varphi®\tilde{j}\oplus\vartheta®\tilde{k}\}} \tag{5.2.1}$$

In addition, it is also important to underline that such an exponent, according to the same symbols adopted, is understood as a *Whole* (see the curly brackets, together with the symbols $\oplus$ and ®).

This means that the corresponding derivative have to be taken with reference to such a *Whole*. Otherwise its corresponding value will be generally underestimated.

If now, for the sake of clarity we synthetically indicate $\{\tilde{\sigma}®\tilde{i}\oplus\tilde{\varphi}®\tilde{j}\oplus\tilde{\vartheta}®\tilde{k}\}=\tilde{\alpha}(t)$, the explicit solution to Eq. (4.1) will result in the form (5.2.2), when it is given in terms of an External Representation. That is, when the coordinates of the various elements of the System are referred to a Reference System of coordinates whose origin is *external* to the System under consideration.

$$\{\tilde{r}\}_s = e^{\begin{Bmatrix} \tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \dots & \tilde{\alpha}_{1n}(t) \\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \dots & \tilde{\alpha}_{2n}(t) \\ \dots & \dots & \dots & \dots \\ \tilde{\alpha}_{n1}(t) & \tilde{\alpha}_{n2}(t) & \dots & \tilde{\alpha}_{nn}(t) \end{Bmatrix}} \tag{5.2.2}$$

The "Matrioska" in eq. (5.2.2) also shows that, as consequence of the Internal Generativity of the System $(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{m}/\tilde{n}\}}$, when the System reaches its Maximum Ordinality, the initial internal structure $(\tilde{m}/\tilde{n})$, as a consequence of the Self-Organization Process, becomes of the form $(\tilde{n}/\tilde{n})$. While the various $\tilde{\alpha}_{ij}(t)$ evidently depend on the initial and boundary conditions, and in the next paragraphs we will show how it is possible to find their explicit expressions.

5.3 Explicit Expression of the Internal Generativity $(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{m}/\tilde{n}\}}$

Let assume that, under the conditions previously described, the explicit expression of the Ordinality $\{\tilde{m}/\tilde{n}\}$, in Eq. (3.2), equals

$$\{\tilde{N}/\tilde{N}\} = \{k, (\tilde{N}/\tilde{N})\} \qquad (5.3.1).$$

Eq. (4.1) then becomes

$$(\underline{\tilde{d}/\tilde{d}\,t})_s^{\{\tilde{N}/\tilde{N}\}} e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{11}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{21}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{N1}(t)\end{pmatrix}, \begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{12}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{22}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{N2}(t)\end{pmatrix} \begin{pmatrix}\cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\end{pmatrix} \begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{1N}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{2N}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \overset{[\rightarrow}{=} \{\tilde{0}\} \qquad (5.3.2),$$

where the symbol " $\overset{[\rightarrow}{=}\{\tilde{0}\}$ ", as previously anticipated, represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (4.1);

- while the symbol " $\overset{[\rightarrow}{=}$ " indicates that the System, during its *Generative Evolution*, is persistently "adherent" to its "origin and habitat" conditions.

5.4 The Initial and Boundary Conditions

Given the particular structure of Eq. (5.3.2), it is possible to directly explicit the term $\overset{[\rightarrow}{=}\{\tilde{0}\}$ in exponential form, so that it can be written as follows

$$e^{\left\{\begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{11}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{21}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{N1}(t)\end{pmatrix}, \begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{12}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{22}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{N2}(t)\end{pmatrix} \begin{pmatrix}\cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\end{pmatrix} \begin{pmatrix}(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{1N}(t)\\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{2N}(t)\\ \cdots\cdots \\ (\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \overset{[\rightarrow}{=} e^{\left\{\begin{pmatrix}\tilde{\beta}_{11}(t)\\ \tilde{\beta}_{21}(t)\\ \cdots\cdots \\ \tilde{\beta}_{N1}(t)\end{pmatrix}, \begin{pmatrix}\tilde{\beta}_{12}(t)\\ \tilde{\beta}_{22}(t)\\ \cdots\cdots \\ \tilde{\beta}_{N2}(t)\end{pmatrix} \begin{pmatrix}\cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\\ \cdots\cdots\end{pmatrix} \begin{pmatrix}\tilde{\beta}_{1N}(t)\\ \tilde{\beta}_{2N}(t)\\ \cdots\cdots \\ \tilde{\beta}_{NN}(t)\end{pmatrix}\right\}} \qquad (5.4.1),$$

which shows that its explicit solution can be obtained by solving *N x N* corresponding differential equations of the form

$$(\underline{\tilde{d}/\tilde{d}\,t})^k \tilde{\alpha}_{ij}(t) = \tilde{\beta}_{ij}(t) \qquad (5.4.2).$$

5.5 Explicit Solution to Eq. (5.4.1), understood in terms of External Description

Equation (5.4.1) generally present an *explicit solution*. This is because in the majority of the most frequent Self-Organizing Systems (both "*non-Living*", "*Living*" and "*Conscious*" Systems), the general structure of the initial conditions can be assumed as being equal to

$$\beta_{ij}(t) = (a_{ij} + b_{ij} \cdot t)^p \qquad (5.5.1),$$

in which *q* can also be a fractional number.

Such initial conditions always lead to the explicit solution of any unknown $\tilde{\alpha}_{ij}(t)$ that appears in Equation (5.4.1). This is because by considering the general definition of the cardinal "Incipient Derivative" (3.2.1), we have that

$$\frac{\tilde{d}^k}{\tilde{d}\,t^k} f(t) = \left(\frac{\tilde{f}'(t)}{f(t)} \right)^k \cdot f(t) = \beta(t) \qquad (5.5.2),$$

in which $\beta(t)$ now represents the initial condition for the generic function $f(t)$.

Consequently, through successive formal passages we have

$$f(t)^{1-k} \cdot f'(t)^k = \beta(t) \qquad (5.5.3),$$

from which

$$f(t)^{\frac{1-k}{k}} \cdot f'(t) = \beta(t)^{\frac{1}{k}} \qquad (5.5.4),$$

whose integral

$$\int_0^t f(t)^{\frac{1-k}{k}} \cdot \tilde{f}'(t) \cdot dt = \int_0^t (\beta(t))^{\frac{1}{k}} \cdot dt \qquad (5.5.5),$$

leads to

$$f(t)^{1/k} \cdot k = \int_0^t (\beta(t))^{\frac{1}{k}} \cdot dt \qquad (5.5.6),$$

and, consequently, we have

$$f(t) = 1/k \cdot \left\{ \int_0^t (\beta(t))^{\frac{1}{k}} \cdot dt \right\}^k \qquad (5.5.7),$$

where $f(t)$ now represents any $\alpha_{ij}(t)$, while $\beta(t)$ represents the corresponding associated initial condition $\beta_{ij}(t)$.

The explicit solution of the generic $\alpha_{ij}(t)$ is then given by

$$\alpha_{ij}(t) = \frac{1}{k} \cdot \left\{ \int_0^t (\beta_{ij}(t))^{\frac{1}{k}} \cdot dt \right\}^k = \frac{1}{k} \cdot \left\{ \int_0^t ((a_{ij} + b_{ij} \cdot t)^p)^{\frac{1}{k}} \cdot dt \right\}^k = \frac{1}{k \cdot b_{ij}} \cdot (\frac{k}{p+k} \cdot (a_{ij} + b_{ij} \cdot t)^{\frac{p}{k}+1})^k \qquad (5.5.8).$$

However, the explicit solution to the First Fundamental Equation (4.1) *does not end here*.

In fact, the General Solution to Eq. (4.1) is characterized by an *additional contribution*. That is, an "Emerging Solution" that correspondently shows an "*Emerging Property*" of the Self-Organizing Systems: the *Diffusive Generativity* among the various elements of the same System, which gives origin to the *Harmony Relationships*.

5.6 The Harmony Relationships

The *Process of Genesis* of the *Harmony Relationships* can be shown by adopting two different *descriptive modalities*, that is: by adopting an External Representation or, alternatively, an Internal Representation.

The two Representations are substantially equivalent between them. However, the adoption of an Internal Representation is able to Ostend much more clearly the abovementioned "Excess of Quality" on behalf of the System analyzed.

This is because, as already anticipated, an External Representation is the one in which each element of the System is referred to a system of coordinates characterized by an origin which is external to the System analyzed. Whereas, in the case of an Internal Representation, the various elements of the System are referred to a system of coordinates which is internal to the System analyzed.

In latter case, each element $\tilde{\alpha}_{ij}(t)$ of the System is preferably referred to the corresponding element the main diagonal belonging the same row *i*, and this leads to the following Representation

$$\{\tilde{r}\}_s = e^{\begin{Bmatrix} 0 & \tilde{\alpha}_{12}(t) & \dots & \tilde{\alpha}_{1N}(t) \\ \tilde{\alpha}_{21}(t) & 0 & \dots & \tilde{\alpha}_{2N}(t) \\ \dots & \dots & \dots & \dots \\ \tilde{\alpha}_{N1}(t) & \tilde{\alpha}_{N2}(t) & \dots & 0 \end{Bmatrix}} \qquad (5.6.1),$$

in which all the elements of the main diagonal are evidently equal to zero, whereas all the other elements $\tilde{\alpha}_{ij}(t)$ assume a binary-duet structure, and thus satisfy the following *Specularity* Relationships

$$\{\tilde{\alpha}_{ij}(t)\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \{\tilde{\alpha}_{ji}(t)\}^{\{\tilde{2}/\tilde{2}\}} \qquad (5.6.2),$$

which represent a much more profound concept than the traditional symmetry (in fact the symbol "$\overset{*}{=}$" does not represent an equality, but a simple *assignation condition*).

Such a Representation then allows us to show the Generative Process that leads the System to its Maximum Ordinality and, at the same time, to its Maximum Stability conditions, because it *restructures the internal relationships* between the various elements in such way as these show an additional "*emerging*" *property*, which is *initially* based on the following "topological" Relationships:

$$\tilde{\lambda}_{12} \oplus \tilde{\alpha}_{12}(t) \overset{*}{=} \tilde{\lambda}_{1j} \oplus \tilde{\alpha}_{1j}(t) \qquad \text{for } j = 3, \dots\dots N \qquad (5.6.3)$$

together with all their associated *incipient* derivatives, up to the order N-1

$$\{\overset{\circ}{\tilde{\lambda}}_{12} \oplus \overset{\circ}{\tilde{\alpha}}_{12}(t)\}^{\tilde{k}} \overset{*}{=} \{\overset{\circ}{\tilde{\lambda}}_{1j} \oplus \overset{\circ}{\tilde{\alpha}}_{1j}(t)\}^{\tilde{k}} \qquad \text{for } k = 1, \dots\dots N-1 \qquad (5.6.4),$$

where $\tilde{\lambda}_{ij}$ represent their corresponding *internal reciprocal* Correlating Factors, which are clearly distinct from the values of the initial conditions, because the latter are already included in the correlative $\tilde{\alpha}_{ij}(t)$.

Such properties represent *the bases* of the previously mentioned Property of *Diffusive Generativity*, which is faithfully represented by the following Harmony Relationships

$$\{\tilde{\alpha}_{1,j+1}(t) \oplus \tilde{\lambda}_{1,j+1}(t)\} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\} \qquad \text{for } j=1,2,3,\dots\text{N-1} \qquad (5.6.5),$$

whose explicit *Process of Genesis* is illustrated in Appendix 1, while the associated Ordinal Roots of Unity $(\sqrt[N-1]{\{\tilde{1}\}})_j$ are illustrated in Appendix 2.

If we now take into account the Harmony Relationships (5.6.5), together with their *specific structure* and the *correlative symbology* adopted, the Solution to the First Fundamental Equation pertaining to the System analyzed can be represented as follows

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\} \circ \left\{ (\sqrt[N-1]{\{\tilde{1}\}})_1, (\sqrt[N-1]{\{\tilde{1}\}})_2, \dots\dots\dots\dots, (\sqrt[N-1]{\{\tilde{1}\}})_{N-1} \right\}} \qquad (5.6.6),$$

which reflects the Self-Organization of the Systems in terms of "*couples*", according to an *Internal Description* and, at the same time, it shows that the basic "*topological*" structure of the reference couple "12" (see Eq. (5.6.3)) has been correspondently "transformed" and "updated", as a consequence of the Diffusive Generative Process which leads to the Harmony Relationships.

6. Explicit Solution to the Two Fundamental Equations of the M.O.P, understood as *a Whole*

The M.O.P., considered in its two Fundamental Equations understood *as a Whole*, differently from the problems formulated in TDC, always presents an *explicit solution*. This is especially due to IDC and, in particular, both to the solution to the First Fundamental Equation in the form of Matrioska and the associated Harmony Relationships, which allow to represent the System in the form of "couples", by assuming one *arbitrary couple* of elements as a reference.

So that, precisely because of such specific characteristics, the M. O. P. enabled us to reconsider and explicitly solve some "particular" problems, generally dealt with in literature in terms of TDC, which are generally considered as being "unsolvable", "intractable", "with drift". The solutions of which ended up by showing that the Maximum Ordinality Principle has an extremely general validity (Giannantoni, 2014, 2016).

The *Explicit Solution* to the Two Fundamental Equations of the M.O.P, understood as a Whole, can be obtained by introducing the solution to the First Fundamental Equation (4.1), previously shown,

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{(\sqrt[N-1]{\{\tilde{1}\}})_1,(\sqrt[N-1]{\{\tilde{1}\}})_2,\ldots\ldots\ldots,(\sqrt[N-1]{\{\tilde{1}\}})_{N-1}\right\}} \quad (5.6.6),$$

into the Global Feed-Back Process represented by the Second Fundamental Equation (4.2), which so transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{B}(t)\}\circ\left\{(\sqrt[N-1]{\{\tilde{1}\}})_{13},(\sqrt[N-1]{\{\tilde{1}\}})_{14},\ldots\ldots\ldots,(\sqrt[N-1]{\{\tilde{1}\}})_{1N}\right\}} \quad (6.1),$$

where

$$\tilde{B}(t) = \left\{\begin{pmatrix}\oplus\,\tilde{A}(t)\\ \Theta\,\tilde{A}(t)\end{pmatrix},\begin{pmatrix}\Theta\,\tilde{A}(t)\\ \oplus\,\tilde{A}(t)\end{pmatrix}\right\} \quad (6.2)$$

and

$$\tilde{A}(t) = \{\{\tilde{\alpha}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}}\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}\}^{(\tilde{2}/\tilde{2})} \oplus \oplus \ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\}) \quad (6.3),$$

in which the term $\ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\})$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.

This latter contribution is particularly important for *the System stability*, especially when the System interacts with another System of its surrounding Habitat.

Equation (6.1), together with Eqs. (6.2) and (6.3), then represents *the Explicit "Emerging Solution" to the Maximum Ordinality Principle*, formulated in two "Incipient" Differential Equations ((4.1) and (4.2)), when the latter are properly considered *as being a Whole*.

7. General Validity of the Explicit Solution to the Maximum Ordinality Principle

Equation (6.1), considered with the associated Eqs. (6.2) and (6.3), has a *general validity* because, at the same time, it is *valid* not only for *non-Living* Systems, but also for *Living* Systems and *Human* Systems too.

What's more, the same fact that solution (6.1) is *always an Explicit Solution* represents a very general property that evidently has a huge relevance from an *operative* point of view.

In addition, Solution (6.1) introduces some further fundamental novelties of *gnoseological nature*, which enabled us to clearly assert that "The "Emerging Quality" of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (M.O.P.), offers a *Radically New Perspective to Modern Science*" (Giannantoni, 2016).

This is exactly what also suggested a possible of reformulation of such a Solution into a corresponding version formulated in operative terms.

8. Explicit Solution to the M.O.P. reformulated *in operative terms* by means an EQS Simulator

In order to have an explicit solution that may result much easier to program on a computer and, in particular, on a simple PC, the previous Explicit Solution can be restructured in more *operative terms*, in order to realize an "Emerging Quality Simulator" (EQS), which, however, is not "equivalent" to a traditional computer program.

This is because, even if conceived for *operative finalities*, EQS always *keeps memory* of the genetic Ordinality of the Processes analyzed. So that the various forms of Ordinality, although considered in operative terms, they will always be accounted for in terms of their "correlative associated cardinalities".

If we then suppose for example that the *Relational Space* of the System is represented by the following three generative coordinates $\{\tilde{\sigma},\ \tilde{\varphi},\ \tilde{\vartheta}\}$, that are characteristic of a "non-Living" System, the fundamental Relationships of EQS are shown here below:

a) $$\tilde{\rho}_{1j}(t_0) = \tilde{A}\cdot e^{\tilde{S}_l(t_0)} \quad (8.1) \qquad \text{where} \qquad \tilde{S}_l(t_0) = \psi_1\cdot E_l\cdot[B_l\cdot\tilde{\Sigma}_0 - C_l\cdot(\tilde{\Phi}_0+\tilde{\Theta}_0)] \quad (8.1.1)$$

b) $$\tilde{\varphi}_{1j}(t_0) = \psi_1\cdot E_l\cdot[B_l\cdot\tilde{\Phi}_0 - C_l\cdot\tilde{\Sigma}_0] \qquad (8.2)$$

c) $$\tilde{\theta}_{1j}(t_0) = \psi_1\cdot E_l\cdot[B_l\cdot\tilde{\Theta}_0 - C_l\cdot\tilde{\Sigma}_0 + C_l(\tilde{\Phi}_0-\tilde{\Theta}_0)] \qquad (8.3)$$

where $$B_l = \cos(\sqrt{2}\cdot\psi_l) \qquad C_l = D_l = \frac{1}{\sqrt{2}}\sin(\sqrt{2}\cdot\psi_l) \qquad (8.4)$$

and $$\psi_l = \psi_2\cdot\frac{\varepsilon_2+2\pi\cdot l}{N-1} \qquad (8.5),$$.

in which:

i) $\tilde{\Sigma}_0, \tilde{\Phi}_0, \tilde{\Theta}_0$ synthetically represent the Ordinal coordinates of the *reference couple*, arbitrarily chosen, generally termed as "*couple 12*", considered at the time t_0. So that the symbols $\Sigma_0, \Phi_0, \Theta_0$ synthetically stand for $\{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\}$. Such coordinates, however, in transient conditions, and always *in operative terms*, become respectively

$$\Sigma_0(t) = \Sigma_0(t_0) + \overset{\circ}{\Sigma}_0(t)\cdot k\cdot\Delta t \qquad (8.6)$$

$$\Phi_0(t) = \Phi_0(t_0) + \overset{\circ}{\Phi}_0(t)\cdot k\cdot\Delta t \qquad (8.7)$$

$$\Theta_0(t) = \Theta_0(t_0) + \overset{\circ}{\Theta}_0(t)\cdot k\cdot\Delta t \qquad (8.8),$$

where $\overset{\circ}{\Sigma}_0(t), \overset{\circ}{\Phi}_0(t), \overset{\circ}{\Theta}_0(t)$ are the values of the "Incipient" Derivatives which, step by step, at any time t, correspond to the trend of the coordinates $\Sigma_0(t), \Phi_0(t), \Theta_0(t)$ evolving in time, while k indicates a given number of time steps Δt each time considered;

ii) the Ordinal factor $\psi_1 \cdot E_l$ originates from the assumption that in the Harmony Relationships, here reproduced for the sake of clearness

$$\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \qquad (5.6.5),$$

for j=1,2,3,….N -1

the terms $\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}$, after a previous *reduction of the Ordinality* $\{\tilde{2}/\tilde{2}\} \to 1$, can be directly expressed in terms of a *specific periodicity* $E_l = \dfrac{\varepsilon_1 + 4\pi \cdot l}{N-1}$ (8.9), which, at the same time, is modulated by the Ordinal factor ψ_1;

iii) Then, after having rewritten the Ordinal Relationships in the following form

$$Exp\{\tilde{\sigma}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\} \overset{*}{=} Exp[(\sqrt[N-1]{\tilde{1}})_l \circledR \{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\}] \qquad (8.10)$$

iv) and after having assumed the explicit expression of the *Ordinal Roots of Unity*, illustrated in Appendix 2 (Eqs. (A2.5) and (A2.6)) and here explicitly recalled for the sake of clarity

$$(\sqrt[N-1]{\tilde{1}})_l \overset{*}{=} Exp\{\tilde{\alpha} \circledR \tilde{i} \oplus \tilde{\beta} \circledR \tilde{j} \oplus \tilde{\gamma} \circledR \tilde{k}\} \qquad (A2.5),$$

where
$$\alpha = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \qquad \beta = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \qquad \gamma = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1} \qquad (A2.6),$$

the expansion series of Eq. (A2.5), together with the contextual adoption of the Rules of the Ordinal Product (5.1.3), (5.1.4), (5.1.5), leads to the Ordinal Relationships (8.1), (8.1.1), (8.2), (8.3), initially introduced, with the associated coefficients given by Eqs. (8.4), (8.5), (8.9).

For the sake of completeness, it is worth adding that:

- The symbol $\{\tilde{1}\}$ represents *the Unity of the System* (understood as *a Whole*) by means the representation of the *Unity* of its *Proper Space of Relations*

- $\varepsilon_1, \varepsilon_2, \varepsilon_3$ characterize the spatial orientation of the System as a Whole, with reference to its Ordinal Proper Space and, more specifically, with reference to the Reference "Couple 12"

- in Eq. (8.9) the "periodicity" of the "spinor" $\tilde{i}$ is assumed equal to 4π, because it is expressed in steradians

- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radians, because these spinors are always "orthogonal", both among them and with respect to the spinor $\tilde{i}$. An "orthogonality" that can be seen as a form of reciprocal "irreducibility" (as also indicated by the same Relational Products);

- while the Factor "$\tilde{A}$" represents an *Internal Ordinal Factor* according to which all the *radial* "Uniances" of the various Couples are appropriately referred to the *radial* "Uniance" of the Reference Couple "12". This latter concept will clearly be illustrated in a next section specifically titled "*Distance and Uniance*".

At the same time, by means of the *Internal Ordinal Factor* "$\tilde{A}$", the cardinalities "associated" to the various "Uniances" are all expressed in terms of a desired scale of measure.

9. General Considerations on the Explicit Solution reformulated in *operative terms* by EQS

From the previous exposition, it should result as being evident that the *Harmony Relationships* (further illustrated in Appendix 1) represent an "Irreducible Excess". That is an "Exceeding" manifestation of the *Generativity of the System*, where the latter is at the same time *Self-Organizing*, of *Ordinal Nature*, and understood as *a Whole*.

This means that *the same Explicit Solution* reformulated in *operative terms*, precisely because obtained through an Ordinal Deductive Process from the Harmony Relationships and the Ordinal Roots of Unity (further illustrated in Appendix 2), represents an "*Emerging Solution*" from the Maximum Ordinality Principle. Consequently, even if the single Relationships refer to each single couple "*1j*", and thus to the three "distinct" variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, the latter do not represent a simple traditional "vector", but an "Ordinal vector". That is a *unique and sole* Relational Entity, which is usually represented in *curly brackets*, such as $\{\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}\}$, precisely because it is understood as *a Whole*.

This means that the three variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, although recognizable as being "distinct", they are not conceptually "separable" between them.

Such an assertion is also even truer (and in particular way) with reference to the *various triples of variables* pertaining to *all the couples which compose* the System, which *a fortiori* are not conceptually "separable" between them precisely because the System is understood *as a Whole*.

In other words, the Fundamental Relations pertaining to EQS previously shown do not only furnish the *N-1* single *Ordinal vectors* $\{\tilde{\rho}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\}$ that characterize each single couple of the System, but they also represent, even more, a *Unified* Ordinal description of the System understood as Whole.

In other terms, the coordinates furnished by the *Operative Solution* are not conceptually "separable" between them, *neither with reference to each single couple*, *nor* with reference to *all the various couples* of the System as a Whole.

This leads us to point out another important aspect.

9.1 Distance and "Uniance"

A direct and correlative consequence is that, even if at a "preliminary and intuitive" interpretation such Ordinal Relationships could be thought as giving the "distances" between the various couples of the System analyzed, in reality, in adherence to the M.O.P, such an interpretation (and the corresponding "terminology") should be substantially modified. In particular, by adopting a more appropriate term, such as "*Uniance*", instead of that of "distance".

This is because the concept of "*distance*" tends more to *divide*, than to *unify*. In fact, the same *etymology* of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

Consequently, in an Ordinal Perspective the term "distance" should preferably be replaced by a different term, possibly able to indicate the concept of "*union*" of two elements, more than their "distance".

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" perspective represents a "*dis-tance*", in an Ordinal Perspective indicates a "*uni-ance*". That is, it indicates that the two elements form "one sole thing" of *Ordinal Nature*, precisely because they are the Exit of the same Generative Process. So that the term "Uniance" expresses an *Ordinal concept*, and not a mere cardinal concept, such as that of "distance". Any "Uniance", in fact, is characterized by its own Ordinality.

As a simple example, let us think of a couple of elements $\tilde{\alpha}_{ij}(t)$ whose Relationship is characterized by a *Binary-Duet* Ordinality $\{\tilde{\alpha}_{ij}(t)\}^{\{\tilde{2}/\tilde{2}\}}$. Such a specific and proper *Ordinality* is precisely that which represents *the Ordinal* "*Unity*" between two elements of the System. While, at the same time, its "associated cardinality" only indicates their topological distribution in the Relational Space of the System.

Consequently, when all the various "Uniances" are considered in the context of the Harmony Relationships, they reveal that the System is a *Whole of Ordinal Nature*, in perfect adherence to the Maximum Ordinality Principle.

In addition, such an assertion has also an *even more general sense*, that is: it is precisely the *Generativity* of the Self-Organizing System the one which, with its proper *Diffusivity*, characterizes all the elements of the System in terms of "Ordinal Relationships". In that sense, such Ordinal Relationships are all of *genetic nature*, like in the case of "brothers".

In fact, as previously anticipated, "brothers" are termed as such not because of their "direct reciprocal relationships", but because of their *direct reference* to *the same genetic principle*: their father (or their mother or both).

Consequently, in perfect "*Adherence*", the term "*Uniance*" *synthesizes* the concept of an *Ordinal Unity of Genetic Nature*.

9.2 Proper Space and Proper Time

Another important aspect that has to be underlined is precisely that synthetically indicated in the title.

In fact the Maximum Ordinality Principle shows that Each Self-Organizing System, precisely because characterized by its own "Emerging Quality", evolves in a "*time*" and a "*space*" which are *exclusive and specific* of each System analyzed. Consequently, the latter can be more faithfully termed as "*Proper Time*" and "*Proper Space*" of the System (Giannantoni, 2018).

This is an aspect that is radically different from the case of the Traditional Scientific Approach, in which *time* and *space* are assumed as being *absolute*.

Such a difference, however, *does not represent a real "obstacle*" with specific reference to the interpretation of the output of EQS Simulator. What is important, in fact, is to know that such a "difference" exists and, at the same time, to be aware of their correlative *different Nature*. In this case, in fact, such a "difference" can always be dealt with in perfect analogy with the "reduction" of the *Uniances*, when the latter have to be compared with the correlative *distances*.

Such a "difference", in addition, is so specific and characteristic of the Self-Orgnizing Systems, that it cannot even be "reduced" to the *space-time* conception of General Relativity.

In fact, as shown in (Landau & Lifchitz, 1966), the General Relativity introduces the concept of "*space contraction*" and "*time dilatation*" between two reference systems in a reciprocal movement, according to the following relationships

$$\Delta x' = \Delta x \cdot \sqrt{1 - V^2 / c^2} \qquad (9.2.1) \qquad\qquad \Delta t = \frac{\Delta t'}{\sqrt{1 - V^2 / c^2}} \qquad (9.2.2).$$

It is then possible to show (Giannantoni, 2018) that Einstein's "*space-time conception*" represents a *particular modality* at introducing the concept of the *second order* "*incipient*" *derivative*. Such a particular modality, however, manifests by itself at *a simple cardinal level*, corresponding to a "reduction" process of the *Proper Space* and *Proper Time* of a given System.

This means that Einstein's *space-time conception* in reality corresponds to the introduction of the *second order* "*incipient*" *derivative*, considered, however, at its mere "*cardinal level*".

10. Two "*com-possible*" Scientific Approaches, albeit "not equivalent" between them

The two above mentioned Scientific Approaches, with their corresponding formal languages, TDC and IDC, respectively, when considered with reference to their corresponding "presuppositions" (that is the subjacent "way of thinking") result as being two different descriptive modalities which are always "*com-possible*". In the sense that they *do not exclude each other*. They simply *co-exist*.

This is because, as already anticipated, the Traditional Scientific Approach, which leads to TDC, *cannot exclude* (in principle) the adoption of a different mental categories and their corresponding formal language (e.g. IDC), because of the *absence* in its presuppositions (especially "necessary" logic) of any form of *perfect induction*.

On the other hand, the same happens in the case of the adoption of IDC, precisely because of the *same reason*, although the latter is based on mental categories characterized by a different form of Logic (e.g. the "Generative" Logic).

Consequently, the two formal languages, TDC and IDC, can *always* be adopted independently from one another. Although this "com-possibility" does not mean that they are "equi-valent" between them.

Their "in-equivalence", in fact, can easily be shown by comparing the different *consequences* of their respective adoption, when such consequences are obviously considered in the light of their corresponding "mental categories".

In fact, *beside* the Traditional Scientific Approach, which affirms that "Every System is a *mechanism*" (at a phenomenological level), there is also the possibility of a different Approach, according to which "*Every System is a Self-Organizing System*" (always at a phenomenological level). This is the fundamental reason why they lead to the adoption of *two corresponding different formal languages*, with some associated important consequences.

In the first case, in fact, the adoption of TDC leads to:

i) *Unsolvable Problems* (in explicit formal terms);

ii) *Intractable Problems* (even by adopting the most advanced computers);

iii) *Problems characterized by experimental "drifts*", which always represent an indication of possible "side effects";

iv) In addition, it is worth pointing out that TDC can present some "side effects" even in the case of accurate experimental confirmations. Such "side effects", in fact, can result as being "*masked*" by the same fact that all the experimental confirmations are always based on the adoption of *methods, instrumentation and measurements* that are conceived (and designed) in a perfect conformity with the fundamental presuppositions of TDC (Giannantoni, 2016).

Vice versa, the adoption of IDC does not present such problems, whereas, in turn, it presents several advantages.

In fact, as already anticipated, the adoption of IDC is finalized to describe the "Emerging Quality" of "Self-Organizing Systems". This leads to the formulation of the M.O.P., which is able to offer *a radically New Perspective* to Modern Science. That is: "*Every System is a Self-Organizing System*" (see Tab. 1).

This is because IDC results as being the most appropriate language able to describe the fundamental characteristics of "Self-Organizing Systems". In fact, the "Incipient Differential Calculus" (IDC):

i) is able to represent, in appropriate formal terms, the "Emerging Quality" of Self-Organizing Systems as an "*Irreducible Excess*";

ii) In this way TDC enabled us to formulate a very general Principle, the Maximum Ordinality Principle (M.O.P.), which can be understood as "*One Sole Reference*" Principle (Giannantoni, 2014);

iii) The latter in fact results as being valid *in any field* of analysis (from *non-living* Systems, to *living* Systems and *human social* Systems too);

iv) In addition, the adoption of IDC *always* leads to *explicit formal solutions*;

v) At *any topological scale* (e.g. from atoms to galaxies);

vi) Both *under steady state and variable conditions*;

vii) What's more, the corresponding Solution to *any* mathematical model based on the M.O.P. (and thus formulated in terms of IDC) always results as being an "*Emerging Solution*". That is, a Solution whose *Ordinal Information content* is always much higher than the Ordinal content corresponding to the initial formulation of the problem;

viii) As a direct consequence, this leads to the fact that any "Emerging Solution" can never be reduced to mere "functional relationships";

xi) This is also means that the adoption of IDC *does not require any* specific reference to the traditional Physical Laws or to the well-known Thermodynamic Principles (precisely because the latter are always understood as "functional relationships");

xv) Finally, the adoption of IDC never leads to "side effects". This is because, even when an "Emerging Solution" might manifest some related "Emerging Exits" (Giannantoni, 2016), the latter can always be interpreted as being corresponding "Extra Benefits", initially not recognized as such. This leads to point out another fundamental aspect.

11. More general *in-equivalence* between the Two Scientific Approaches, especially with reference to the relationships between Man and the Environment

Although from a general point of view the *in-equivalence* between the two formal languages can preliminarily be recognized at the level of "Thinking", such an in-equivalence is even much more marked at the level of "Decision Making and Acting". Especially when considering, as a basic reference criterion, the corresponding different concepts of "inter-relationships" between Man and the Environment (Giannantoni, 2016).

This is because the adoption of TDC always "reflects" the general idea that "every system is a *mechanism*", while the "com-possible" formal language IDC is always orientated at describing any system as a "*Self-Organizing System*". This is the fundamental reason for the adoption of the three *new mental categories* (shown in Tab. 1), which are radically different from the three basic presuppositions of the former.

This easily leads to recognize that the most profound "in-equivalence" between TDC and IDC situates at the level of Decision Making and Acting. In fact:

i) At the level of "Decision Making" the two formal languages will evidently lead to make decisions (that will become consequential future *actions*) in a perfect *conformity* with their respectively different way of thinking: TDC, in conformity with its "*aprioristic*" presuppositions; IDC, vice versa, in conformity with the *new mental categories* that, on the contrary, are adopted "*a posteriori*".

Consequently, in both cases the two formal languages will suggest "decisions" in perfect *conformity* with their corresponding concepts of "*surrounding habitat*": understood as a "*set of mechanisms*", in the case of TDC or, respectively, as "*a unique Self-Organizing System*" in the case of IDC (Giannantoni, 2016);

ii) At the level of Action, however, it is exactly *where* it is possible to recognize the most marked differences between the two Scientific Approaches. This is because, in such a case, the specific different *origin* of each formal language, together with the associated *powerful expressive capacity that any formal language is able to manifest,* represent the fundamental aspects that systematically "guides" (sometimes even "forces") the research for specific *practical solutions* to the various problems and their subsequent actual implementation.

In other terms, the profound differences between the two Scientific Approaches, characterized by their corresponding formal languages, TDC and IDC, respectively, become particularly evident at the "level of Action", because the corresponding formal solutions *become consequential facts* (Giannantoni & Zoli, 2010).

In this respect, the Ostensive Examples previously considered in the various Biennial Emergy Conferences (from 2010 to 2020), are sufficiently clear to show the profound differences that may result, *in practice*, when adopting the one or the other descriptive formal language.

In addition, in the next paragraph we will analyze an ulterior and more radical form of *in-equivalence*.

12. Radical *In-equivalence* between Falsification and Relaunce

Another aspect that points out even more clearly the *in-equivalence* between the Traditional Approach and the Ordinal Approach is the fact that the first one is characterized by "*confirmation/falsification*" processes whereas the second one is characterized by "*Emerging Exits*".

The "confirmation" processes, in fact, are strictly necessary in the case of Traditional Theories, which are adopted "*a priori*", and are specifically based on those mental categories previously recalled. In particular, *necessary logic*.

At the same time, the absence of experimental confirmations of the corresponding conclusion of Traditional Theories, represents a valid argumentation for their "*falsification*" (according to Popper's Falsification Principle).

On the contrary, the Ordinal Approach based on the "Emerging Quality" of Self-Organizing Systems, strictly speaking *does not require* correlative "confirmation processes" in order to be accepted as being a "valid" Approach.

This is because the Ordinal Approach is adopted "*a posteriori*", that is downstream the recognition of the *Manifestation* of the Quality as an "Irreducible Excess", and consequential adoption of the new renewed correlative Mental Categories.

So that, the research for the "*Maximum Adherence*" of the correlative Over-Deductions (in Generative Logic) to experimental results, does not represent, properly speaking, the research for a "confirmation". But, paradoxically, it represents the "confirmation" of a "*denial*". Or better, "a confirmation" that can be termed as being "*not less than*".

In fact it is exactly such circumstance the one that properly generates the concept of *Relaunce*.

The latter in fact consists in recognizing that the description of the "Emerging Quality", as performed at a preliminary given stage, if characterized by "*Emerging Exits*", can be cognized as being "not less than". Thus the description can be re-proposed at a *Higher Level of Ordinality* with respect to the one initially supposed and assumed to describe the Process (or Phenomenon) analyzed.

At this stage, the profound "*in-equivalence*" previously shown between the two formal languages, which mainly and clearly manifests at the level of "facts", may suggest, as a possible conclusion, the consideration of an extremely important question: "where are we going", as a consequence of the adoption of *one* or *the other* descriptive formal language": TDC or IDC?

Conclusion. Where are we going?

The afore-mentioned differences between the two Scientific Approaches and their correlative formal languages, TDC and IDC, which can preliminarily be recognized at a gnoseological level and, even more, at the level of their respective *practical* consequences, enable us to draw some general conclusions that can be synthetically summarized as follows.

From a general point of view, in fact, it is possible to delineate three possible answers to the previous question: i) Modern Science is so radically rooted in TDC (and in its corresponding presuppositions) that it is extremely improbable to hypothesize, in spite of the afore-mentioned intrinsic *limitations* of such a formal language, a rapid change of the corresponding paradigm. In this sense, we have to expect a generalized persistence in the adoption of the traditional formal approach (TDC);

ii) this fact, however, does not prevent from thinking that, occasionally, and with reference to specific problems, some Scientists will decide to *exclusively* adopt the innovative IDC approach;

iii) more probably, because of the afore-mentioned "*com-possibility*" between TDC and IDC, it may be expected the adoption of both formal approaches *at the same time*, so as to choose the optimal *operative* solutions on the basis of the corresponding experimental results.

By always taking into account, however, that TDC translates, in formal terms, a "*self-referential*" gnoseological approach, while IDC represents, always in formal terms, a "*hetero-referential*" gnoseological approach (as previously illustrated and synthetically summarized in Tab. 1).

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Appendix 1. Process of Genesis of the Harmony Relationships

In this Appendix we want to point out, in more explicit terms, what synthetically asserted in the text, that is: the Harmony Relationships represent, by themselves, an "*Emerging Solution*" which, in addition, is also "*Exceeding*" with respect to the Solution to the First Fundamental Equation.

In fact, what we presented at paragraph 5.6 are nothing but the *basic presuppositions* for the formulation of the Harmony Relationships, which, however, do not represent a "*necessary consequence*" of those presuppositions, because they manifest an "*Extra*", or better, an "*Irreducible Excess*" with respect to them.

Let us thus recall the basic elements that will enable us to show that the Harmony Relationships precisely represent an "Emerging Extra" of *Generative Nature*.

We have seen in fact that the Emerging Solution to the First Fundamental Equation allow us to write the following *topological* "Assignation Relationships"

$$\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \{\tilde{\alpha}_{1j}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{1j}(t)\}^{\{\tilde{2}/\tilde{2}\}} \quad \text{for} \quad j = 3,4,...N \qquad \text{(A1.1)},$$

while the corresponding *topological* "Assignation Relationships", written in terms of "Incipient" Derivatives assumed the form

$$\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\}^{\tilde{k}} \overset{*}{=} \{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\}^{\tilde{k}} \qquad \text{for} \qquad k = 1,2,......N-1 \qquad \text{(A1.2)},$$

in which, for simplicity of notation, the Ordinalities $\{\tilde{2}/\tilde{2}\}$, which appear in Eq. (A1.1), are thought as being included in the symbols of the quantities to which they refer to.

More specifically, Eqs. (A1.2) cannot be interpreted as a "necessary consequence" of Eqs. (A1.1), because the latter are obtained on the basis of "Incipient" Derivatives. Consequently, they are all of *Generative Nature*.

In fact, if rewritten in the following form

$$\frac{\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\}^{\tilde{k}}}{\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\}^{\tilde{k}}} \overset{*}{=} \tilde{1} \qquad \text{for} \qquad k = 1,2,......N-1 \qquad \text{(A1.3)},$$

they allow to assert that the considered System is already characterized by a proper and specific "*Interior Unit*", of *Generative Nature,* formally represented by the symbol "$\tilde{1}$".

Such a "Unity", however, is still in the form of "*Not Less Than*". This is because:

- in a Generative Contest, they are certainly not *the parts* that, through the Relationships "*between*" them, give "Origin" to the "Excess of Unity"
- because it is exactly true the opposite: in fact, it is the *Generative Unit* of the System that, with its *proper* "*Excess*", *Qualifies* the Relationships "*between*" *the parts*.

Consequently, the most Adherent Formulation of the Self-Organizing Generative Process is that which can be obtained by re-proposing Eqs. (A1.3) in the form

$$\frac{\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\}^{\tilde{k}}}{\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\}^{\tilde{k}}} \overset{*}{=} \{\tilde{1}\} \qquad \text{per } \forall k \qquad \text{(A1.4)},$$

or better, even more properly, as follows

$$\frac{\{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\}}{\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\}} \overset{*}{=} \{\tilde{1}\}^{\frac{1}{(N-\tilde{1})}} \qquad j = 2,......N \qquad \text{(A1.5)},$$

in which the symbol $\{\tilde{1}\}$ now formally represents the *Generative Whole*, which, at the same time, is *Self-Organizing* and of *Ordinal Nature*. While its *unique* and *sole* exponent $1/(N-\tilde{1})$ explicitly represents the fundamental concept previously anticipated, that is: it is the "Whole", with its *proper* Generative "*Excess*", the one that properly "*Qualifies*" the Relationships "*Between*" *the parts*.

This is obviously true not in the sense of Relationships understood "two by two", but as the specific Reflex of an Ordinal Unit, which, in any case, represents an "Irreducible Excess" with respect to the simple "composition" of the single "parts".

Consequently, Relation (A1.5), can also be written in the form

$$\{\overset{\circ}{\tilde{\alpha}}_{1j}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{1j}\} \overset{*}{=} \{\tilde{1}\}^{\frac{1}{\{\tilde{N-1}\}}} \circ \{\overset{\circ}{\tilde{\alpha}}_{12}(t) \oplus \overset{\circ}{\tilde{\lambda}}_{12}\} \qquad \text{for} \quad j = 2, \ldots\ldots N \qquad \text{(A1.6)},$$

which, reinterpreted in terms of "*Progenitor Relationships*", finally leads to the formal expression of the Harmony Relationships. The latter, written in the form

$$\{\tilde{\alpha}_{1,j+1}(t) \oplus \tilde{\lambda}_{1,j+1}(t)\} \overset{*}{=} (\sqrt[\tilde{N-1}]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\} \qquad \text{for} \quad j = 1,2,\ldots.N-1 \qquad \text{(A1.7)},$$

clearly show that the *Diffusive Generativity* "updates", by Assignation, the *same reference couple* "12".

Eqs. (A1.7) then clearly show that all the elements of the Ordinal Matrix (5.6.1) can be obtained on the basis of *one sole couple* $\tilde{\alpha}_{ij}(t)$ assumed as reference and N-1 associated Correlating Factors.

In this respect, it is also worth noting that condition (A1.2) is properly that which represent the fundamental presupposition of what could be termed as an *Intensive Whole*, precisely because of the "*consonance*" between all the generative derivatives up to the order N-1.

This is the specific reason why, by means of the M. O. P., and its correlative Harmony Relationships, it was possible to reconsider some "particular" problems that, in the Traditional Scientific Literature, are generally known as being "*unsolvable*", "*intractable*", "*with drift*". Whose solutions ended up by showing that the Maximum Ordinality Principle has an extremely general validity (Giannantoni C., 2014).

Appendix 2. The Ordinal Roots of Unity $\{\tilde{1}\}$

In this respect it is worth observing that Relationships (A1.7) are written in such a form only for reasons of clarity and exposition simplicity. In such a form, in fact, it could seem that the various elements that characterize the System are "still" related, "between" them, according to Relationships of the type "two by two".

In reality, if one makes explicit the term $(\sqrt[\tilde{N-1}]{\{\tilde{1}\}})_j$ according to its more specific meaning, that is as $\{\tilde{1}\}^{\frac{1}{\{\tilde{N-1}\}}} \equiv \{\tilde{1}\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}}$, in which *N*-1 refers to the cardinality, while $(\tilde{N-1})$ refers to the Internal Ordinal (N-1)-ary Relationship, it is possible to more appropriately write (by pointing out the Ordinalities $\{\tilde{2},\tilde{2}\}$, previously underwritten)

$$\{\tilde{\alpha}_{1j}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{1j}(t)\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \{\tilde{1}\}^{\frac{1}{\{N-1,(\tilde{N-1})\}}} \circ \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \qquad \text{(A2.1)},$$

that is, even more explicitly, in the form

$$\{\tilde{\alpha}_{1,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{1,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \overset{*}{=} \begin{pmatrix} (\sqrt[N-1]{\{\tilde{1}\}})_1 \\ (\sqrt[N-1]{\{\tilde{1}\}})_2 \\ \\ (\sqrt[N-1]{\{\tilde{1}\}})_{N-1} \end{pmatrix} \circ \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \qquad \text{(A2.2)},$$

from which it is possible to recognize that the single “cardinal” values that in Eq. (A1.7) appear as they were “distinct”, and, in addition, as being “separated”, in reality they are the *Reflex of an Ordinal Unit* that transcends them, and relates them in the form of an *(N-1)-ary* Relationship.

This is the aspect that (more than others) clearly manifests that the Harmony Relationships represent an “Excess” with respect the initial Assignation Relationships (5.6.3) and (5.6.4).

As far as the “explicit” meaning of the Ordinal Routs of Unity is concerned, previously synthetically indicated in the form

$$(\sqrt[N-1]{\{\tilde{1}\}})_j \qquad \text{per} \quad j=1,2,3,\ldots N-1 \qquad \text{(A2.3)},$$

it is worth expressly pointing out that the symbol $\{\tilde{1}\}$ represents the *Unity of the System* (understood as a *Whole*), with specific reference to the *Unity of its Proper Space* (as well as its *Relational Space*).

Such a Fundamental Unit can be then expressed by the following Relationship

$$\{\tilde{1}\} = e^{\{\alpha \circledR \tilde{i} \oplus \beta \circledR \tilde{j} \oplus \gamma \circledR \tilde{k}\}} \qquad \text{(A2.4)}.$$

Consequently, the Ordinal Roots $(\sqrt[N-1]{\{\tilde{1}\}})_l$ will be represented in the following form

$$\{\tilde{1}\}_l = e^{\frac{\{\alpha \circledR \tilde{i} \oplus \beta \circledR \tilde{j} \oplus \gamma \circledR \tilde{k}\}}{N-1}} \qquad \text{(A2.5)},$$

where:

- $\tilde{i}, \tilde{j}, \tilde{k}$ are the fundamental spinors of the Relational Space, understood in their more general sense, that is, as the specific foundation of any given System
- α, β, γ are respectively equal to

$$\alpha = \varepsilon_1 + \frac{4\pi \cdot l}{N-1} \qquad \beta = \varepsilon_2 + \frac{2\pi \cdot l}{N-1} \quad \text{e} \quad \gamma = \varepsilon_3 + \frac{2\pi \cdot l}{N-1} \qquad \text{(A2.6)},$$

- where the “periodicity” of the “spinor” $\tilde{i}$, as we already know, is equal to 4π , because expressed in *steradians*;
- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to 2π radiants (each), because these spinors are always “orthogonal”, both between them, and with respect to the spinor $\tilde{i}$ (an orthogonality that can be understood, inter alia, as a form of reciprocal “irreducibility”);
- the quantities $\varepsilon_1\ \varepsilon_2\ \varepsilon_3$ represent specific “parameters” of the *Relational Space* each time considered, with specific reference to the “couple 12”.

Sometimes (for example in the case of Protein Folding), for an easier “topological” representation Eqs. (A2.6) can also represented as

$$\frac{\alpha}{N-1} = \frac{\varepsilon_1 + 4\pi \cdot l}{N-1} \qquad \frac{\beta}{N-1} = \frac{\varepsilon_2 + 2\pi \cdot l}{N-1} \qquad \frac{\gamma}{N-1} = \frac{\varepsilon_3 + 2\pi \cdot l}{N-1} \qquad \text{(A2.7)},$$

which however can always re-proposed in the previous form (A2.6) through an appropriate choice of the parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3$.

On the basis of the previous exposition, it should be even clearer that the Harmony Relationships represent an "*Irreducible Excess*", that is an "*Exceeding*" Manifestation of a *Generative System*, which, at the same time, is *Self-Organizing*, of *Ordinal Nature*, and, above all, is understood as *a Whole from the very beginning*, and not vice versa.