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The Evolution of the Universe as a Self-Organizing System in the light of the Maximum Ordinality Principle, in the absence of "dark energy" and "dark matter"

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ABSTRACT

- The main aim of this paper is to show that the Evolution of the Universe, when the latter is modeled as a Self-Organizing System in the light of the Maximum Ordinality Principle, can appropriately be described in explicit terms, without the introduction of the concepts of "dark Energy" and "dark Matter". In addition, its time origin can be better estimated on the basis of its Proper Time and Proper Space.
- To this purpose, we first recall two Ostensive Examples, that is the Solar System and our Galaxy, already described in the past as Self-Organizing Systems in the light of the Maximum Ordinality Principle (MOP), as an introduction to the description of the more general case of the Entire Universe.
- In fact, the concepts that will be pointed out in the case of the two above mentioned Self-Organizing Systems, will enable us to give a different description of the Universe, in particular (but not only), with respect the correlative absence of "dark Energy" and "dark Matter".

INTRODUCTION. ORIGIN OF THE CONCEPT (OR HYPOTHESIS) OF "DARK ENERGY"

- The concept (or hypothesis) of "dark Energy" substantially originates from the fact that, in Astronomy, there are several intrinsically unsolvable problems.
- The case of the Solar System is a particularly meaningful Ostensive Example because, at present, we do not have a mathematical model able to give, in terms of an explicit solution, the topological distribution of the various Planets.
- Such a distribution, in fact, is only approximately described by the semi-empirical Bode's Law, which, however, has never found a physical foundation, neither in Classical Mechanics nor in General Relativity.
- The basic reason of both these two aspects fundamentally resides on the formal limitations due to the famous *"Three-Body Problem"*, demonstrated as being *intrinsically unsolvable* by H. Poincaré (in 1889). A problem which, in turn, is a direct consequence of the fact that both the mentioned theoretical approaches tend to research for a possible solution in mere *functional* terms.
- In this respect, in fact, it is worth recalling that the same H. Poincaré explicitly pointed out that this result is due to the fact that "The conservation of Energy is a limitation imposed on the *freedom* of complex systems." (Poincaré, 1952, p. 133). In other terms, Energy conservation excludes *the emergent novelty* that grows out of complex interactions (see also Mirowski, 2000, p. 5).
- This means that we should research for a Model of the Solar System by assuming that "Energy is not constant".
- Consequently, *an additional contribution* to Energy is required to the solution of a given problem. An *additional contribution* that, however, has an *unknown nature*. Nonetheless, it represents the basis for the concept of "dark Energy", which can be "useful" to model the Solar System as being characterized by a "variable Energy".

Under these conditions, it is also easy to understand why the same modality of description, always researched in functional term, can lead, at the scale of the Universe, to the concept of "Dark Energy".

THE SOLAR SYSTEM IN THE LIGHT OF THE MAXIMUM ORDINALITY PRINCIPLE (MOP)

If vice versa the Solar System is modeled as a Self-Organizing System in the light of the MOP, the distribution of the Planets can easily be obtained in explicit formal terms as already shown in (Giannantoni 2014a), and in more details both in (Giannantoni & Rossi, 2014b, par. 3.2) and in (Giannantoni 2016, 2017 cap. 3, 2018).

The detailed analysis of the Solar System, developed in the previous references, it is here recalled in its fundamental results, with specific reference to the concept of "dark Energy".

In the case of the Solar System, in fact, the corresponding *Harmony Relationships* can be written as follows (Giannantoni 2018)

$$\tilde{\{\rho_{1,l+1}, \phi_{1,l+1}, \tilde{\theta}_{1,l+1}\}}_{t}^{\tilde{2}\tilde{2}\tilde{2}} = (\sqrt[1-1]{1})_{j} \otimes \tilde{\{\rho_{12}, \phi_{12}, \tilde{\theta}_{12}\}}_{t}^{\tilde{2}\tilde{2}\tilde{2}} \text{ for } j=1,2,3,\dots,11-1$$
(1)

where, for simplicity of notation, the *habitat conditions* are considered as being already "included" in the

topological ordinal variables $\{\rho, \varphi, \vartheta\}$.

- If we now assume, for example, that the couple "12" corresponds to the couple Sun-Mercury, Eqs. (1) give the description of the *Generative Evolution* of the Solar System and, in particular, they give the distribution, at *any time*, of the Ordinal "*distances*" of the Planets (ib.).
- Equations (1), in fact, represent a *unique* "Emerging Solution" which, precisely because of its *higher information content* with respect to the initial formulation of the problem (Giannantoni 2017, ch. 1), is able to describe the "*Emerging Quality*" of the Solar System, when understood as a "Self-Organizing System".
- This is also the basic reason why the distribution of the Planets, although researched for by Classical Mechanics and General Relativity, cannot be get when the Solar System is modeled in mere "functional terms".
- In an Ordinal Description, vice versa, there is no "necessity" of "dark Energy", because in such a context there is "no necessity" of the same "concept of Energy" which is, by itself, a *functional concept*.
- The concept of Energy, in fact, has its specific sense in the case of a "functional description" of the Solar System, but it is not properly required to describe the Solar System as a Self-Organizing System. In other terms, when the relationships between the various Planets are of *generative nature* (ib.).

Something similar happens in the case of "dark Matter".

THE ORIGIN OF THE CONCEPT OF "DARK MATTER"

The concept (or hypothesis) of "dark Matter" originates from another type of unsolvable problems: the distributions of the angular velocities of the Stars in the Galaxies.

- In fact, the non-Keplerian distribution of velocities inside any Galaxy led Scientists to suppose the presence of a "non-visible" matter (thus termed as "dark") that could explain such an *unexpected behavior*. This is because the general tendency of the Traditional Approach (as previously said) is that of "reducing" any new "effect" to "efficient and functional causes".
- Vice versa, if any given Galaxy is thought of as being a "Self-Organizing System", and it is modeled on the basis of the MOP, it is rather easy to show that, apart from a very limited area near the center of the Galaxy, we always have that

$$\rho_{1j}(t) \cdot \varphi_{1j}(t) \cong \cos t \tag{2},$$

where $\rho_{1j}(t)$ and $\varphi_{1j}(t)$ represent the "distance" and the angular velocity, respectively, of any given star (j) with reference to star "1", that is the nearest one to the "Center" of the Galaxy (Giannantoni 2017, 2018).

Equation (2) then shows that there is no "necessity" to introduce the hypothesis of "dark Matter". The distribution of the stars in a Galaxy, in fact, manifests an "*Irreducible Excess*" that cannot be interpreted in terms of a common

Keplerian trend represented by

$$\rho_{1j}(t) \cdot \overset{\circ}{\varphi}_{1j}(t) \cong \cos t / \sqrt{\rho_{1j}(t)}$$
(3).

The difference between Eq. (2) and Eq. (3), however, is able to explain the "subjacent" reasons that led Scientists to formulate the hypothesis of "dark Matter".

At the same time the two Ostensive Examples here recalled are also able to show the wide "flexibility" of the MOP in modelling the "Emerging Quality" of "Self-Organizing Systems" ranging from 11 bodies (the Solar System), up to $50 \div 100 \cdot 10^9$ Stars of a Galaxy (see also Giannantoni 2014a).

"DARK ENERGY" AND "DARK MATTER" AT THE LEVEL OF THE UNIVERSE

- On the basis of the previous Ostensive Examples it is rather easy to understand the reasons why the entire Universe is thought as being constituted of much more matter than that observable and also characterized by energy "in excess".
- In 1933 the Swiss Astronomer Fritz Zwicky explicitly declared that, in actual fact, there was more matter in the Universe than that we are able to observe.

- The Astronomers termed this matter as "Dark Matter", because the latter does not emit any electromagnetic radiation. A concept that assumed a renewed importance when in '70 the Astronomer Vera Rubin attributed to this enigmatic matter the possible explanation of the movements and the velocities of the Stars.
- Consequently, Scientists have devoted many resources to identify the "Dark Matter" in the space, but without any success.
- In 1998 a team of American and Australian Astro-Physicists discovered the expansion of the Universe, characterized by its associated "dark Energy", and they obtained the Nobel Price for Physics.
- However, in spite of the enormous resources implied, no one theory or observation has been able to define what is such a sort of energy.
- In short, both "Dark Energy" and "Dark Matter" remain as two "mysteries" on which Astronomers have investigated for decades.
- In "parallel" to the previous approach, or better, as a "com-possible" approach, in this paper we want to show that, if the Universe is described as a Self-Organizing System on the basis of the Maximum Ordinality Principle, its corresponding evolution can always be obtained in explicit terms, in the form of an "Emerging Solution", without "Dark Energy" and "Dark Matter".

THE UNIVERSE AS A SELF-ORGANIZING SYSTEM IN THE LIGHT OF THE MOP

- The description of the Universe, as a Self-Organizing System, can be given by adopting the general formulation of the Maximum Ordinality Principle (MOP), already presented at the last Biennial Emergy Research Conference (Giannantoni 2018), which is here re-proposed for the sake of clearness.
- The Maximum Ordinality Principle (MOP), whose verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", is formulated by means of two fundamental equations, which are so *strictly related to each other*, so as to form *a Whole* (Giannantoni 2010, 2012, 2014a,b, 2016, 2017, 2018):

The First Fundamental Equation

It is formulated as follows

$$(\underline{\widetilde{d/dt}})_{s}^{(\widetilde{m/n})}\{\widetilde{r}\} \stackrel{[\rightarrow]}{=} \{\widetilde{0}\}$$
(4)
$$(\widetilde{m/n}) \to Max \to \{\widetilde{2/2}\} \uparrow \{\widetilde{N/N}\}$$
(4.1)

where: $\{r\}$ is the *Relational* Space of the System under consideration, while (m/n) represents its corresponding

Ordinality, which reaches its *maximum* when it equals $\{2/2\} \uparrow \{N/N\}$ (as indicated in (4.1)). In this respect it is worth noting that:

- i) the underlined symbol $(\underline{d/dt})_s$ explicitly indicates that the Generative *Capacity* of the System (more appropriately termed as *Generativity*), is "*internal*" to the same System, precisely because it is the one which gives origin to its Self-Organization as a Whole;
- ii) the symbol " $=\{0\}$ " represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact, it now represents, at the same time:
- the specific "origin and habitat" conditions associated to the considered Ordinal Differential Equation (4);

- while the symbol "=" indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

The Second Fundamental Equation

It is formulated as follows

$$(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}\otimes (\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\} \stackrel{i\rightarrow}{=} \{\tilde{0}\}$$
(5),

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System.

Equation (5), in fact, formally asserts that the *Relational Space* of the System $\{r\}$, which "emerges" as a solution from the First Equation, interacts (in the form of the product \otimes) with *its proper Generative Capacity*

 $(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$, so as to originate a *comprehensive* Generative Capacity which, *at any time*, is always adherent to the origin and habitat conditions of the Second Fundamental Equation.¹

General Explicit Solution to the two fundamental Equations understood as a Whole

Equation (4) always presents an *explicit solution* which can always be written in the following general form

$$\tilde{\{r\}} = e^{\tilde{\{\alpha(t)\}}} = e^{\left[\left(\begin{array}{c} \tilde{\alpha}_{11}(t) \\ \tilde{\alpha}_{21}(t) \\ \cdots \\ \tilde{\alpha}_{N1}(t) \end{array} \right) \left(\begin{array}{c} \tilde{\alpha}_{12}(t) \\ \tilde{\alpha}_{22}(t) \\ \cdots \\ \tilde{\alpha}_{N2}(t) \end{array} \right) \left(\begin{array}{c} \tilde{\alpha}_{1N}(t) \\ \tilde{\alpha}_{2N}(t) \\ \cdots \\ \tilde{\alpha}_{NN}(t) \end{array} \right) \right] }$$
(6)

where the *Relational* Space $\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}}$ depends on the Nature of the System analyzed, while the corresponding structure of each term of the Ordinal Matrix depends on the Specific Generativity $(\tilde{d}/\tilde{d}t)_{e}$.

For example, if the *Relational* Space of the System is characterized by the three topological coordinates $\{\sigma, \varphi, \vartheta\}$, which are always considered as *the exit of a Generative Process*, we have that

$$\{\tilde{r}\}_{s} = e^{\tilde{a}(t)} = e^{\{\tilde{\sigma}\otimes\tilde{i}\oplus\tilde{\varphi}\otimes\tilde{j}\oplus\tilde{g}\otimes\tilde{k}\}}$$
(6.1),

because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_{s} = \{\tilde{\rho}^{\mathbb{R}}\tilde{i}^{\mathbb{R}}e^{\tilde{\varphi}^{\mathbb{R}}\tilde{j}}^{\mathbb{R}}e^{\tilde{g}^{\mathbb{R}}\tilde{k}}\} = \{e^{\tilde{\sigma}^{\mathbb{R}}\tilde{i}}^{\mathbb{R}}e^{\tilde{\varphi}^{\mathbb{R}}\tilde{j}}^{\mathbb{R}}e^{\tilde{g}^{\mathbb{R}}\tilde{k}}\} = e^{\{\tilde{\sigma}^{\mathbb{R}}\tilde{i}\oplus\tilde{\varphi}^{\mathbb{R}}\tilde{j}\oplus\tilde{g}^{\mathbb{R}}\tilde{k}\}} = e^{\tilde{\alpha}(t)}$$
(6.2).

Equation (6) thus describes the *Generative Evolution* of the System as the exit of an Ordinal Cooperation of N Co-Productions and their associated N Inter-actions. At the same time, when the Process has reached its Maximum

Ordinality, each term $\alpha_{ij}(t)$ of the Ordinal Matrix (as we will see later on) is represented by a binary-duet Relationship $\{\tilde{\alpha}_{ij}(t)\}^{\tilde{2}/2}$, although in the Ordinal Matrix (6) it is represented in terms of $\tilde{\alpha}_{ij}(t)$ only for the sake of notation simplicity.

At the same time, the adoption of the brackets "{}" in Eq. (6) is explicitly finalized to remind us that the Ordinal Matrix represents a mathematical concept understood as a *Whole*.

In fact, all the elements of the Ordinal Matrix (in Eq. (6)) satisfy the following "Ordinal Relationships"

$$\{\alpha_{i,j+1}(t)\}^{\{\tilde{2}\tilde{2}\tilde{2}\}} \oplus \{\lambda_{i,j+1}(t)\}^{\{\tilde{2}\tilde{2}\tilde{2}\}}\} \stackrel{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_{j} \otimes \{\alpha_{12}(t)\}^{\{\tilde{2}\tilde{2}\tilde{2}\}} \oplus \{\lambda_{12}(t)\}^{\{\tilde{2}\tilde{2}\tilde{2}\}}\}$$
(7)
for $j=1,2,3,\dots,N-1$

where the additional terms $\{\lambda_{i,j}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ explicitly account for the associated *habitat conditions*. Eqs. (7) can also be termed as "Harmony Relationships" precisely because they show that all the elements $\{\alpha_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\lambda_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ of the Ordinal Matrix can be obtained by means of *one sole* arbitrary couple $\{\alpha_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\lambda_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, assumed as reference, and the N-1 Ordinal Roots $(\sqrt[N-1]{\{\tilde{1}\}})_j$ of Unity $\{\tilde{1}\}$. Consequently, if each element of the Ordinal Matrix (in Eq. (6)) is expressed in terms of the reference couple $\{\alpha_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\lambda_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, the solution to Eq. (4) assumes the form

¹ The symbol \otimes represents a generalized form of the "vector" product expressed in terms of *spinors* (see Giannantoni 2010a).

$$\tilde{\{\alpha_{12}(t) \oplus \tilde{\lambda}_{12}(t)\}} \circ \left\{ \begin{pmatrix} (N - \sqrt{1}) \\ (N -$$

where, always for the sake of simplicity, the term $\{\alpha_{12}(t) \oplus \lambda_{12}(t)\}$ stands for $\{\alpha_{12}(t)\}^{\{2/2\}} \oplus \{\lambda_{12}(t)\}^{\{2/2\}}$. The same Ordinal Matrix, in addition, may be represented in a synthetic form by means one sole symbol, when adopting the following synthetic notation

$$\{ \left(\sqrt[N-1]{\tilde{1}} \right)_{ij} \}^{\uparrow \{\tilde{N}/\tilde{N}\}}$$
(9),

where the arrow " \uparrow " explicitly reminds us that the Ordinality $\{N/N\}$ has always to be considered as being a particular form of *Over-Ordinality*.

In this way the explicit solution to Eq. (4) can more synthetically be expressed as follows

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ(\sqrt[N-1]{1})^{\uparrow\{\tilde{N}/\tilde{N}\}}}$$
(10).

Consequently, if such a solution is now introduced into the Global Feed-Back Process represented by Equation (5), it transforms the latter into a typical Riccati's Equation of Ordinal Nature, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{B}(t)\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}}$$
(11),

where

$$\tilde{B}(t) = \left\{ \begin{pmatrix} \oplus \tilde{A}(t) \\ \Theta \tilde{A}(t) \end{pmatrix}, \begin{pmatrix} \Theta \tilde{A}(t) \\ \oplus \tilde{A}(t) \end{pmatrix} \right\}$$
(12)

and

$$\tilde{A}(t) = \{\{\tilde{\alpha_{12}}(0)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda_{12}}(0)\}^{\{\tilde{2}/\tilde{2}\}}\} \circ (\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}\}^{(\tilde{2}/\tilde{2})} \oplus \ln(\tilde{c_1} \oplus \{\tilde{c_2}, t\})$$
(13),

in which the term $\ln(c_1 \oplus \{c_2, t\})$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process.

Equation (11) then represents the Explicit "Emerging Solution" to the Maximum Ordinality Principle, formulated in two "Incipient" Differential Equations ((4) and (5)), considered as being a Whole.

General Validity of the Explicit Solution (11)

Equation (11), considered with the associated Eqs. (12) and (13), has a *general validity* because, at the same time, it is *valid* for *non-living* Systems, *Living* Systems and *Human* Systems too (Giannantoni 2018).

What's more, the same fact that Solution (11) is *always an explicit solution* represents a very general property that evidently has a huge relevance from an *operative* point of view.

- In addition, Solution (11) introduces some further fundamental novelties of *gnoseological nature* (as we will see later on), which will enable us to clearly illustrate the concept already anticipated in (Giannantoni 2016, 2018), that is: "The "Emerging Quality" of Self-Organizing Systems, when modeled according to the Maximum Ordinality Principle (MOP), offers a *Radically New Perspective to Modern Science*".
- In this respect, we can compare the Traditional Description of the Universe with its possible Description as a Self-Organizing System.

COMPARISON BETWEEN THE TWO DIFFERENT APPROACHES TO THE DESCRIPTION OF THE UNIVERSE

- Such a comparison, at least at a preliminary stage, could be performed on the basis of the evolution of a given number of observable Galaxies.
- For example, it could be possible to consider the first 1000 Galaxies observable from the Earth, and then compare their evolution with the data already available by means of Hubble telescope.
- This surely represents a preliminary comparison because on the basis of the most recent estimations, always referable to Hubble telescope, the number of Galaxies could probably be 2 trillion, that is about 10 times as much the previous estimations.

- In this respect, we can surely affirm that, in spite of the fact that such a comparison is theoretically feasible, it would require a very long time, and also not negligible computation difficulties, especially when there are not available explicit formal solutions.
- Nonetheless, even in the presence of the above-mentioned difficulties, on the basis of the differences between the two approaches, it is surely possible to affirm that:

i) The Traditional Approach, which is based on *efficient causality, necessary logic, functional relationships* (Giannantoni 2104a, b, 2015, 2016), in order to get an appropriate description of the phenomena is "forced" to hypothesize, *a posteriori*, a *quantitative* "excess" of both "Energy" and "Mass". The latter, however, although conceived and understood as being "Physical Entities", do not present any form of confirmation at the level of a proper and corresponding Physical Nature;

ii) The Ordinal Approach, vice versa, based on *generative causality, adherent logic, ordinal relationships* (ib.), from the very beginning of the description already has the concept of "Excess", understood, in this case, as an the "Excess of Quality". This is because the Ordinal Approach is finalized to describe Self-Organizing Systems, always characterized by an "Emerging Quality", understood as an "Irreducible Excess" of *Generative Nature*. This, in fact, is the real basic reason for the radical change of the correlative abovementioned mental categories.

- This enables us to anticipate, precisely because of these reasons, that we may expect a "marked" difference as far as the *time origin* of the Universe is concerned, with respect to the case in which the latter is estimated on the basis on the "Dark Energy" and "Dark Matter". This is because, according to the Maximum Ordinality Principle, Self-Organizing Systems always evolve in their Ordinal *Proper Time* (Giannantoni 2018), and, in addition, such an evolution is also influenced by the evolution of the corresponding Ordinal *Proper Space* (ib.).
- At this stage, on the basis of the previous exposition, we could ask: what is the "best" Approach?
- We can surely answer that the two Approaches *always co-exist* and their validity is always based, as usual, on experimental confirmations. In all cases, however, the two Approaches are *always possible*, or better, "*compossible*", because they do not present any reciprocal interference, or, even, any reciprocal exclusion.
- In this respect we can specifically assert that, According to the Ordinal Approach, based on the Maximum Ordinality Principle, it is impossible to give a "con-*clusive*" answer to the question, because in the Ordinal Approach the Concept of "Quality" *is intrinsically enable to "close*". "Quality", in fact, is always "Emerging". Or better, it is always "Over-Emerging".

Nonetheless, the previous exposition can lead us to a form of an "Ordinal Con-clusion", which, in reality, is an "Over-Conclusion", that is, it represents a particular form of "Rebound".

A "CON-CLUSION" AS A "REBOUND"

As already anticipated, such a concept of "Rebound" is perfectly adherent to the MOP, whose formulation is finalized to describe the "Emerging Quality" of Self- Organizing Systems, understood as an "Irreducible Aspect".

In this sense, the previous considerations about the expected difference in the time origin of the Universe, as a consequence of its Ordinal *Proper Time*, and the correlative "influence" of the associated Ordinal *Proper Space*, can lead us to a sort of "Rebound" with respect the previous concepts of Proper Time and Proper Space presented in (Giannantoni 2018). That is: at the level of the Universe (and correspondently, at the level of any Self-Organizing System), the Ordinal *Proper Time* and the Ordinal *Proper Space*, initially considered as being two distinct concepts (ib.), in reality they form *One Sole and Unique Entity*, of *Ordinal Nature*.

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www.ordinality.org: author's website that presents a general framework about the MOP, by starting from the Mathematical Formulation of Odum's Maximum Em-Power Principle up to the Mathematical Formulation of the MOP, together with some Ostensive Examples mentioned in this paper.