# The Accelerated Expansion of the Universe in the light of the Maximum Ordinality Principle in the absence of "Dark Energy" and "Dark Matter"

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## Abstract

*The main aim of the paper is that of presenting (and at the same time offering) a different perspective for the analysis of the accelerated expansion of the Universe. A perspective that can surely be considered as being "in parallel" to those proposed in the past, because it is based on Generative Logic, and thus in the absence of perfect induction. Consequently, the present proposal it is not understood as being "exclusive", and thus it is not specifically in contrast with other approaches that, precisely because based on "necessary logic", are equally characterized by the absence of perfect induction. This means that any perspective of analysis, when confirmed by experimental results, it always keeps its validity from an operative point of view.*

*The basic difference only consists in the fact that the Evolution of the Universe is now modeled by considering the Universe as a Self-Organizing System, which is thus analyzed in the light of the Maximum Ordinality Principle.*

*In such a perspective, the Accelerated Expansion of the Universe, more than due to "Dark Energy" and "Dark Matter", "reveals" as being a direct "Emerging Exit" of the Ordinal Generativity of the Universe, which diffuses its harmonious "effects" on all the various Galaxies (and, at the same time, on all the astronomical bodies included in them).*

## Introduction

As anticipated in the Abstract, it is possible to show that the Accelerated Expansion of the Universe is nothing but the Ordinal "Emerging Exit" of a Generative Process, specific of the Universe as a Whole, understood as a Self-Organizing System, and thus characterized by such a particular "Emerging Quality", as described by the Maximum Ordinality Principle. Such an "Emerging Exit" will be then shown by considering the time evolution of a Galaxy Cluster, made up of about 1000 stars and approximately 10 Mpc in diameter.

To this purpose, it is worth recalling the basic aspects pertaining to the Maximum Ordinality Principle and its subjacent perspective.

## 1. The Physical Principle adopted for the analysis and its correlative formal language based on the "Incipient Derivative"

The Maximum Ordinality Principle (MOP), presented in 2010 at the 6th Biennial Emergy Conference, Univ. of Florida [1], is a Principle that is apt to describe the "Emerging Quality" of Self-Organizing Systems. Its verbal enunciation asserts that "*Every System tends to maximize its Ordinality, including that of its surrounding habitat*", and it is formulated by means of two fundamental equations, which are so *strictly related to each other* so as to form *a Whole* [2].

Its specific introduction is directly referable to the fact that Self-Organizing Systems always show an unexpected "*excess*" with respect to their phenomenological premises. So that they usually say: "*The Whole is much more than its parts*".

Such an "excess" can be termed as *Quality* (with a capital Q) because it cannot be understood as being a simple "property" of a given phenomenon. This is because it is *never reducible* to its phenomenological premises in terms of traditional mental categories: *efficient causality*, *logical necessity*, *functional relationships*. Consequently, it cannot be described in terms of the *traditional derivative* that, at the level of formal language, represents the perfect reflex of such "a priori" mental categories.

This evidently suggests a *radically new* gnosiological perspective and, *in adherence*, the adoption of "*new mental categories*"[1]: *Emerging Causality*, *Generative Logic*, *Ordinal Relationships*. These, in turn, suggest the development of a completely *new formal language*, in order to formulate *one sole Reference Principle* [3], the Maximum Ordinality Principle, so that the description of Self-Organizing Systems could possibly result as being faithfully conform to their "Emerging Quality" [4].

In this perspective, the present approach is perfectly *Com-possible* with those more traditional ones, because the *Generative Logic* adopted has not the property of "the perfect induction". Consequently, the present approach cannot be considered as being "exclusive". However, the same happens for all the various approaches based on *Classical Logic*, also termed as *necessary logic*. This means that no approach among those until now adopted can be considered as being "exclusive", because all of them are based on the *necessary logic* previously mentioned. Nonetheless, when confirmed by experimental results, they always keep their validity from an operative point of view.

The *only difference* consists in the fact that in the traditional approaches the reference mental categories are adopted "a priori", while in the present Ordinal Approach they are adopted "a posteriori". In fact, they are based on the previously recognized "*Emerging Quality*" of Self-Organizing Systems.

This is why *a new concept of derivative* was introduced, that is the "*Incipient Derivative*" [5], which is defined as follows

$$(\frac{\tilde{d}}{\tilde{d}t})^{\tilde{q}} f(t) = \underset{\Delta t:0 \to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}t}\right)^{\tilde{q}} \circ f(t) \quad \text{for} \quad \tilde{q} = \tilde{m}/\tilde{n} \tag{1}.$$

A definition which clearly shows that the "*Incipient Derivative*" is not an "operator", like the derivative $(d/dt)$ in the Traditional Differential Calculus (TDC), but it could be termed as a "*generator*", because it describes the *Generativity* of a given Process, *in its same act of being born*. In fact:

i) The sequence of the symbols is now interpreted according to the *direct priority* of the three elements that constitute its definition (from left to right). This is why they acquire a completely different meaning with respect to the traditional one;

ii) The three symbols, in fact, do not represent "three" distinct operations, but a *unique and sole* Generative Process;

iii) The symbol $\tilde{Lim}$, whose etymological origin comes from the Latin word "Limen" (which means a "threshold"), represents the "threshold" of that "ideal window" from which we observe and describe the considered phenomenon;

iv) The symbol $\tilde{\Delta}t : 0 \to 0^+$ now indicates not only the initial time of our registration, but also the proper "*origin*" (in its etymological sense) of *something new* which we observe (and describe) in its proper act of being born, as a Generative Process [6];

v) It is then evident that the "operator" $\tilde{\delta}$ now registers the variation of the observed property $f(t)$, not only in terms of quantity, but also, and especially, in terms of Quality (as the symbol "tilde" would expressly remind). Thus the ratio which appears in Eq. (3) indicates not only a quantitative variation in time, but both the variation in Quality and quantity;

vi) Consequently, when we take the incipient (or "prior") derivative of Ordinality $\tilde{q}$ of any $f(t)$, the *exit* of such a process will keep "memory" of its genetic origin because, besides its quantity, it will result as being structured according the indication of such an exponent. The latter in fact is properly termed as *Ordinality*, because it precisely expresses how each part of the output is *genetically Ordered* to the Whole and, at the same time, *how each part is related to all the others* in terms of *Ordinal Relationships* (as shown by Eqs. (A1) and (A5) in Appendix);

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[1] These "*new mental categories*" can no longer be termed as "pre-suppositions", because they are not defined "a priori" (as in the case of Traditional Approach). In fact, they are adopted "a posteriori", only on the basis of the "Emerging Quality" previously recognized. "*Emerging Causality*", in fact, refers to the *capacity* of a Self-Organizing System to manifest an "irreducible excess"; "*Generative Logic*" correspondently refers to the capacity of our mind to draw "*emerging conclusions*". That is, "conclusions" whose information content is much higher than the information content corresponding to their logical premises, although they are persistently "adherent" to the latter. "Ordinal Relationships", in turn, refer to particular relationships of *genetic nature*, like in the case of "brothers". The latter in fact are termed as such not because of their "direct reciprocal relationships", but because their *direct reference* to *the same genetic principle*: their father (or their mother or both).

vii) In this way the "incipient" derivative represents the *Generativity of the considered Process*, that is the output "excess" (per unit time) characterized by both its Ordinality and its related cardinality, while the sequence of the symbols in its definition (Eq. (1)) can be interpreted as representing a *unique inter-action process* between the same;

viii) The above-mentioned reasons clearly show why the "incipient" derivative is able to *unify* (and, at the same time, to specify) the description of the various Self-Organizing Processes, when explicitly understood in terms of Quality;

ix) This also means that the Incipient Derivative has an *Exit* that is generally different from the *result* of the corresponding derivative in TDC, even when its Ordinality is reduced to *a mere cardinality*. For example, the derivative of order $n$ of the function $e^{\alpha(t)}$, evaluated according to Faà di Bruno's formula, when compared with the corresponding *same order* incipient derivative, respectively give

$$(\frac{d}{dt})^n e^{\alpha(t)} = e^{\alpha(t)} \sum \frac{n!}{k_1! k_2! \dots k_n!} \cdot \left(\frac{\dot{\alpha}}{1!}\right)^{k_1} \left(\frac{\ddot{\alpha}}{2!}\right)^{k_2} \cdot \left(\frac{\alpha^{(n)}}{n!}\right)^{k_n} \tag{2}$$

and

$$(\frac{\tilde{d}}{\tilde{d}t})^n e^{\alpha(t)} \overset{*}{=} e^{\alpha(t)} \cdot [\overset{\circ}{\alpha}(t)]^n \tag{3}$$

where $\overset{\circ}{\alpha}(t)$ represents the first order incipient derivative. And even if in some cases the two derivatives coincide (for instance when $\alpha(t)$ is linear) such a coincidence has to be seen in the light of the symbol $\overset{*}{=}$ in Eq. (3), which reminds us that any incipient derivative is always the *exit* of a *Generative Logical Process* and not of a *necessary* logical process.

## 2. The First Fundamental Equation of the Maximum Ordinality Principle

On the basis of the previous concept of "incipient" derivative, the First Fundamental Equation is formulated as follows

$$(\underline{\tilde{d}/\tilde{d}t})_s^{(\tilde{m}/\tilde{n})}\{\tilde{r}\} \overset{[\rightarrow}{=}\{\tilde{0}\} \tag{4} \qquad\qquad (\tilde{m}/\tilde{n}) \rightarrow Max \rightarrow \{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\} \tag{4.1}$$

where $\{\tilde{r}\}$ is the *Relational* Space of the System under consideration (see Eq. (A2) in Appendix), while $(\tilde{m}/\tilde{n})$ represents its corresponding Ordinality, characterized by $\tilde{m}$ Ordinal Co-productions and $\tilde{n}$ Ordinal Interactions, which reaches its *maximum* when it equals $\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{N}/\tilde{N}\}$ (as indicated in Eq. (1.1)).

In this respect, it is worth noting that:

i) The *underlined* symbol $(\underline{\tilde{d}/\tilde{d}t})_s$ explicitly indicates that the *Generative Capacity* of the System (more appropriately termed as *Generativity*) is "*internal*" *to the same System*. This is because it is precisely that which gives origin to its Self-Organization as a Whole;

ii) The symbol "$\overset{[\rightarrow}{=}\{\tilde{0}\}$" represents a more general version of the simple *figure* "zero", as the latter systematically appears in the traditional differential equations. In fact it now represents, at the same time:

- the specific "*origin and habitat*" *conditions* associated to the considered Ordinal Differential Equation (4);

- while the symbol "$\overset{[\rightarrow}{=}$" indicates that the System, during its Generative Evolution, is persistently "adherent" to its "origin and habitat" conditions.

## 3. The Second Fundamental Equation of the Maximum Ordinality Principle

It is formulated as follows

$$(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\}\circledR(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\} \overset{[\rightarrow}{=}\{\tilde{0}\} \tag{5}$$

and it can be considered as representing a *global* Feed-Back Process of *Ordinal Nature*, which is *internal* to the same System.

Equation (5), in fact, asserts that the *Relational Space* of the System $\{\tilde{r}\}$, which "emerges" as a solution from the First Equation, interacts in the form of the Relational Product $\circledR$ (defined in Appendix) with *its proper Generative Capacity* $(\tilde{d}/\tilde{d}t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}$. In such a way as to originate a *comprehensive* Generative Capacity, which is particular important for the *Ordinal Stability* of the System, especially when the latter interacts with other surrounding Systems understood as being its proper habitat.

The Maximum Ordinality Principle, in its two fundamental equations, always presents an *explicit solution*, which is illustrated in Appendix. In this case, however, the general explicit solution to the MOP is also presented and structured in a more *operative form*, so that is may result as being more directly and easily adopted in analysing the case under consideration.


## 4. The Accelerated Expansion of the Universe

The accelerated expansion of the Universe will be analyzed by considering, as *a case study*, a Galaxy Cluster made up of about 1000 Galaxies and approximately 10 Mpc in diameter. In such a case the Galaxy Cluster, if modeled in the light of

the MOP, should be considered as being a Self-Organizing System of Ordinality $\tilde{1000}/\tilde{1000}$, that is, as clearly shown in Appendix, an Ordinality which is characterized by 1000 Ordinal Co-Productions and 1000 reciprocal Ordinal Interactions. In such a case, however, even in the presence of an explicit solution of the MOP (as given in Appendix), such a general approach would require the adoption of an appropriate computer and, at the same time, a correlative sophisticated computer code.

However, in order to show the process of an accelerated expansion inside such a Galaxy Cluster, it is also possible to limit the analysis to a significantly reduced portion of the same.

For example, it is possible to consider a single "cone" containing 12 Galaxies, with its vertex in the center of the Cluster and 5 Mpc in length, characterized by an amplitude of about 0.15 steradians.

The Ordinal topological distribution of such 12 Galaxies can be obtained on the basis of the Ordinal Relationships illustrated in the Appendix (Eqs. from (A12) to (A16)), here reproduced for the sake of clarity.

a) $\qquad \tilde{\rho}_{1j}(t_0) = \tilde{A} \cdot e^{\tilde{S}_l(t_0)}$ (6) where $\tilde{S}_l(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]$ for $l = 1,2..N\text{-}1$ (6.1)

b) $\qquad \tilde{\varphi}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Phi}_0 - C_l \cdot \tilde{\Sigma}_0]$ (7)

c) $\qquad \tilde{\theta}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Theta}_0 - C_l \cdot \tilde{\Sigma}_0 + C_l(\tilde{\Phi}_0 - \tilde{\Theta}_0)]$ (8)

where $\qquad B_l = \cos(\sqrt{2} \cdot \psi_l) \qquad C_l = D_l = \dfrac{1}{\sqrt{2}} \sin(\sqrt{2} \cdot \psi_l)$ (9)

and $\qquad E_l = \dfrac{\varepsilon_1 + 4\pi \cdot l}{N-1} \qquad \psi_l = \psi_2 \cdot \dfrac{\varepsilon_2 + 2\pi \cdot l}{N-1}$ (10).


Such Relationships, according to the Ordinal Deductive Process shown in Appendix, represent an "*Emerging Solution*" from the Maximum Ordinality Principle. Consequently, even if such Relationships although referred to each single couple "*1j*", and thus expressed with reference to the three distinct variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, the latter do not represent a simple traditional "vector", but an "Ordinal vector". That is a *unique and sole* Relational Entity, which is usually represented as $\{\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}\}$, because understood as a Whole.

This means that the three variables $\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}$, although recognizable as being "distinct", are not conceptually "separable" between them. Such an assertion is even truer (and in especial way) with reference to the triples of variables pertaining to *all the couples which compose* the System, which are not conceptually "separable" between them precisely because the System is understood *as a Whole*.

In other words, the Fundamental Relations previously shown do not only furnish the *N* single *Ordinal vectors* $\{\tilde{\rho}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\}$ that characterize each single couple of the System, but they also represent, even more, a *Unified* Ordinal description of the System understood as Whole.

A direct and correlative consequence is that, even if at a "preliminary and intuitive" interpretation such Ordinal Relationships could be thought as giving the "distances" between couples of Galaxies of the analyzed System, in reality, in adherence to the MOP, such an interpretation (and the corresponding "terminology") should be substantially modified. In particular, by adopting a more appropriate term, such as "Uniance", instead of that of "distance".

This is because the concept of "*distance*" tends more to *divide*, than to *unify*. In fact, the same *etymology* of the word (from Latin "*dis-stant*") indicates that "one element *stays here* and the other one *stays there*" or, equivalently, "*one is here and the other one is there*".

Consequently, in an Ordinal Perspective the term "distance" should preferably be replaced by a different term, possibly able to indicate the concept of "*union*" of two elements, more than their "distance".

In this respect, by introducing a *neologism* (that "rhymes" with the term "distance", but it exactly indicates the opposite meaning), we could say that the same value that in a "functional" perspective represents a "distance", in an Ordinal Perspective indicates a "*uni-ance*". That is, it indicates that the two elements form "one sole thing" of *Ordinal Nature*, because they are the Exit of the same Generative Process. So that the term "Uniance" expresses an *Ordinal concept*, and not a mere cardinal concept, such as that of "distance". Any "Uniance", in fact, is characterized by its own Ordinality.

As a simple example, let us think of a Binary-Duet Ordinality 2/2. Such a specific and proper *Ordinality* is precisely that which represents *the Ordinal* "Unity" between two elements of the System. While, at the same time, its "associated cardinality" only indicates their topological distribution in the Relational Space of the System (defined by Eq. (A2) in Appendix).

Consequently, when all the various "Uniances" are considered in the context of the Harmony Relationships, they reveals that the System is a *Whole of Ordinal Nature*, in perfect adherence with the Maximum Ordinality Principle. In addition, such an assertion has also an *even more general sense*, that is: it is precisely the Generativity of the Self-Organizing System which, with its proper *Diffusivity*, characterizes all the elements of the System in terms of "Ordinal Relationships". In that sense, such Ordinal Relationships are all of *genetic nature*, like in the case of "brothers". In fact, as previously anticipated, "brothers" are termed as such not because of their "direct reciprocal relationships", but because their *direct reference* to *the same genetic principle*: their father (or their mother or both). The term "Uniance" thus *synthesizes* the concept of an *Ordinal Unity of Genetic Origin*.

## 4. Analysis of the case study on the basis of the previous Ordinal Generative Relationships

The Ordinal Relationships previously shown are able to describe the Ordinal distribution of N Galaxies belonging to any selected cone of a Galaxy Cluster. To this aim, it is sufficient to define the values of the characteristic Ordinal parameters pertaining to the Self-Organizing System analyzed, in terms of their "associated" cardinalities.

In this respect, by considering a cone containing for hypothesis N = 12 Galaxies

- by assuming the associated cardinal values of the coordinates of the reference couple $\{\tilde{\sigma}_{12}(t_0)\ \tilde{\varphi}_{12}(t_0),\ \tilde{\vartheta}_{12}(t_0)\}$ at the time $t_0$, now synthetically represented as $\Sigma_0, \Phi_0, \Theta_0$, equal to $\Sigma_0 = 0.02$, $\Phi_0 = 0006$, $\Theta_0 = 0.006$ respectively.

This is because, although such values represent only associated cardinalities, they cannot be assumed as being identically null, otherwise their corresponding Ordinality will be completely "denied";

- the values of $B_l$ and $C_l$ are assumed as corresponding to an internal periodicity $\psi_2 = 0.05$ and $\varepsilon_2 = 0$;

- while the periodicity $E_l$ is characterized by the value of $\varepsilon_1 = 0$ and it is "modulated" by a periodicity $\psi_1 = 16.5$.

On the basis of such assumptions, by adopting an Internal Scale Factor A = 0.2, the previous Ordinal Relationships give the following Ordinal Topological Distribution of the various couples of Galaxies. Such distribution is thus represented in terms of the "associated cardinalities" of their Uniances $\{\tilde{\rho}_{1j}(t_0), \tilde{\varphi}_{1j}(t_0), \tilde{\vartheta}_{1j}(t_0)\}$, at a given time $t_0$, distributed along a cone of about 5 Mpc in length, with its vertex situated at the center of the Galaxy Cluster.

**Table 1 – Ordinal Topological Distribution of the Galaxies**

| Couple | Index $l$ | $B_l$ | $C_l$ | $E_l$ | $\tilde{\rho}_{1j}(t_0)$ [Mpc] | $\tilde{\varphi}_{1j}(t_0)$ [rad] | $\tilde{\vartheta}_{1j}(t_0)$ [rad] |
|--------|-----------|-------|-------|-------|-------------------------------|----------------------------------|------------------------------------|
| 1-2 | 1 | 0.999184 | 0.020192 | 1.14241 | 0.29016 | 0.105393 | 0.105393 |
| 1-3 | 2 | 0.996739 | 0.040368 | 2.28479 | 0.41637 | 0.195021 | 0.195021 |
| 1-4 | 3 | 0.992668 | 0.060511 | 3.42719 | 0.58989 | 0.268368 | 0.268368 |
| 1-5 | 4 | 0.986978 | 0.080604 | 4.56959 | 0.82362 | 0.324951 | 0.324951 |
| 1-6 | 5 | 0.979677 | 0.100632 | 5.71199 | 1.13134 | 0.364308 | 0.364308 |
| 1-7 | 6 | 0.970779 | 0.120577 | 6.85438 | 1.53618 | 0.386161 | 0.386161 |
| 1-8 | 7 | 0.960298 | 0.140425 | 7.99678 | 2.01856 | 0.389678 | 0.389678 |
| 1-9 | 8 | 0.94825 | 0.160157 | 9.13918 | 2.61327 | 0.374944 | 0.374944 |
| 1-10 | 9 | 0.934655 | 0.17976 | 10.2816 | 3.30629 | 0.341453 | 0.341453 |
| 1-11 | 10 | 0.919536 | 0.199215 | 11.4241 | 4.08117 | 0.288948 | 0.288948 |
| 1-12 | 11 | 0.902917 | 0.218508 | 12.5664 | 4.90959 | 0.217161 | 0.217161 |

The values in Table 1 show that there is a progressive increasing of the *radial* Uniances between the various Galaxies, although such a trend, in Tab. 1, is represented in terms of "associated cardinalities".

Nonetheless, this particular aspect could be seen as being a preliminary indication of different recession velocities between the various Galaxies.

At the same time, the values of the angular distributions of the various Galaxies, after an initial progressive increase, gradually tend to decrease. In this respect, it is worth pointing out that the two angular distributions result as being equal between them only because of the assumption of identical initial values pertaining to $\Phi_0$ and $\Theta_0$.

The Accelerated Expansion of Galaxies will now be analyzed in four successive phases, each one characterized by a progressively increasing level of generality.

## 4.1 First Phase of the Analysis

Let us now assume that Galaxy 2 has a recession velocity, with respect to Galaxy 1, which is equal to $\chi$ [Km/sec/Mpc].

This mean that, in a given time interval $\Delta t$, the corresponding *increase of their radial Uniance* (expressed in Mpc) will be the following one

$$\tilde{\rho}_{12}(t_0 + \Delta t) = \tilde{A} \cdot e^{\psi_1 \cdot E_1 \cdot [B_1 \cdot \tilde{\Sigma}_0 - C_1 \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot (1 + \chi \cdot \Delta t) \tag{11}$$

in which, because of the limited time interval $\Delta t$ considered in this analysis, the value of the astronomical recession velocity $\chi$ can be assumed as being substantially constant.

Eq. (11) is particularly interesting because it offers the possibility of a wider and more general interpretation.

In fact, Eq. (11) can also be rewritten in the following form

$$\tilde{\rho}_{12}(t_0 + \Delta t) = \left\{ \tilde{A} \cdot (1 + \chi \cdot \Delta t) \right\} \cdot e^{\psi_1 \cdot E_1 \cdot [B_1 \cdot \tilde{\Sigma}_0 - C_1 \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]}$$

(12).

Such a reformulation, in fact, even if from a "functional" point of view can be considered as being perfectly "equivalent" to Eq. (11), in reality, in an Ordinal Generative *Context* it is not understood as a "functional" equation but, more properly, as being a *Generative Equation*. In such a *Generative* context, the "Translation" of the Internal Scale Factor $\tilde{A}$, expressed by the term $\left\{ \tilde{A} \cdot (1 + \chi \cdot \Delta t) \right\}$ in Eq. (12), do not simply refer to the single couple "12", but it is considered as being referred

*to all the System*. This is because, the definition of the Scale Factor $\tilde{A}$ (as given in Appendix), understood as an Ordinal parameter "*Internal*" to the System, exactly "reflects" the fact that the radial Uniances $\tilde{\rho}_{1j}(t_0)$, given by the Fundamental Relations, do not only refer separately to each single couple of the System, but they represent a *unified Ordinal* description of the System understood as Whole.

Consequently, the same Translation of the Internal Scale Factor $\tilde{A}$ in reality pertains to *all the Couple of Galaxies*, according the following general reformulation, written in analogy to Eq. (11)

$$\tilde{\rho}_{1j}(t_0 + \Delta t) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot (1 + \chi \cdot \Delta t)$$

(13).

If we now take the "Incipient Derivative" of Eq. (13), this is because we are describing a *Generative Process*, we get

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot \chi$$

(14).

where the notation $\overset{\circ}{\tilde{\rho}}_{1j}$, as we already know, represents the first order incipient derivative.

Eq. (14) then shows that each Galaxy has a velocity of recession that is equal to the *product* between the assumed value $\chi$ and its specific Uniance with respect to Galaxy 1.

Eq. (14) then allows us to assert that, if astronomical observations show that the recession velocity is equal to 73.2 Km/sec/Mpc, we can write Eq. (14) in the form

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot 73,2 \ [\text{Km/sec/Mpc}]$$

(15).

In fact, the term $\tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]}$ represents the radial Uniance between *any* given Galaxy and Galaxy 1, where such a radial Uniance, as already known, is expressed in Mpc.

Such result then clearly shows that the accelerated expansion of the Galaxies is nothing but *one sole Harmonious Process*, as a "reflex" the Harmony Relationships, which are all "coordinated" by the Ordinal Generativity of the considered Self-Organizing System, understood as a Whole.

This interpretation, however, represents only the first modality of showing the Accelerated Expansion of the Galaxies.


## 4.2 Second Phase of the Analysis

A more general approach consists in considering the radial translation of the Ordinal parameter $\Sigma_0$. In such a case, being such an Ordinal parameter an exponent of the exponential "*e*", its translation has to be formulated in logarithmical terms, that is

$$\Sigma_0 + \ln(1 + \chi \cdot \Delta t)$$

(16)

and, at the same time, it has to be reduced by the product $\psi_1 \cdot E_l \cdot B_l$, so that it might be referred to the sole couple "12".

In this way we get

$$\tilde{\rho}_{1j}(t_0 + \Delta t) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot (\tilde{\Sigma}_0 + \frac{\ln(1+\chi \cdot \Delta t)}{\psi_1 \cdot E_l \cdot B_l}) - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \tag{17},$$

which can also be rewritten as

$$\tilde{\rho}_{1j}(t_0 + \Delta t) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)] + \ln(1+\chi \cdot \Delta t)} \tag{18}.$$

If we now take the "Incipient Derivative" of Eq. (18), we get

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)] + \ln(1+\chi \cdot \Delta t)} \cdot \frac{\chi}{(1 + \chi \cdot \Delta t)} \tag{19}.$$

However, if the exponential is rewritten in the form

$$e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot e^{\ln(1+\chi \cdot \Delta t)} = e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot (1 + \chi \cdot \Delta t) \tag{20},$$

the previous "Incipient" Derivative (Eq. (19)) gives

$$\overset{\circ}{\tilde{\rho}}_{1j}(t_0) = \tilde{A} \cdot e^{\psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]} \cdot \chi \tag{21},$$

that is, it gives *the same* Ordinal Exit obtained in the previous phase of the analysis.

This approach, however, results as being more appropriate, because, as we will see in the next phase, it allows us to evaluate,

in addition, the correlative trends of the angular phases $\tilde{\varphi}_{1j}(t)$ and $\tilde{\theta}_{1j}(t)$, characteristic of each specific Uniance.


## 4.3 Third Phase of the Analysis

As already anticipated, the approach just shown as phase 2, even if it leads to the same "Exit" of that of phase 1 as far as the radial expansions of the Uniances of the various couples of Galaxies is concerned, it also offers the possibility of a more general perspective.

In fact, in addition to the previous Exit, it is also able to point out the *variations* of the correlative angular phases $\tilde{\varphi}_{1j}(t_0)$

and $\tilde{\theta}_{1j}(t_0)$, evaluated in the same time interval $\Delta t$.

In fact, by introducing the same radial translation of the Ordinal parameter $\Sigma_0$ given by Eq. (16) in to the Equations (7) and (8), we get

$$\tilde{\varphi}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Phi}_0 - C_l \cdot \tilde{\Sigma}_0] - \frac{C_l}{B_l} \cdot \ln(1 + \chi \cdot \Delta t) \tag{22}$$

$$\tilde{\theta}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Theta}_0 - C_l \cdot \tilde{\Sigma}_0 + C_l(\tilde{\Phi}_0 - \tilde{\Theta}_0)] - \frac{C_l}{B_l} \cdot \ln(1 + \chi \cdot \Delta t) \tag{23}.$$

Consequently, by taking into account that the values of the coefficients $C_l$ in Tab 1 tend to progressively increase, while the coefficients $B_l$ tend to progressively decrease, this means that Eqs. (22) and (23) progressively tend toward *zero*.

Consequently, the recession of the various Galaxies almost uniformly tends toward a progressively "*radial*" expansion.

Such an assertion is evidently true only in a short time period, as the one previously considered. In fact, in the long run, such a general trend can lead to an "inversion" of the values of the angular components of the Uniances. In fact Eqs. (22) and (23) can progressively become negative, with respect to their positive values corresponding to the initial conditions, evaluated with reference to the initial values of the Ordinal parameters $\Phi_0$ and $\Theta_0$.

This means that, while the various Uniances continue to progressively expand "radially", their angular components show a progressive "retrograde" trend (with respect to their initial conditions). And even if the "radial" expansion of the Uniances given by Eq. (15) represents the prevailing trend, its combined consideration with the angular components of the Uniances, may contextually represent a *"preliminary" indication* of that very particular phenomenon which is generally termed as *"The Synchrony of Galaxies"*. An aspect that, however, will be analyzed in more detail in a successive paper.

## 4.4 Fourth Phase of the Analysis: the Accelerated Expansion directly referred to the Total Ordinal Uniance expressed as $\{\tilde{\rho}_{1j}, \tilde{\varphi}_{1j}, \tilde{\vartheta}_{1j}\}$

The Generative "Exits" of the previous approach just shown offer an even more general perspective of analysis.

In fact, instead of considering the sole radial recession of the Galaxies, it is also possible to consider their recession velocities (and correlative accelerations) with reference to the Ordinal Uniance understood in its proper sense, that is, in its three Ordinal components, considered as *one sole and unique entity*.

In fact, if we consider the definition of the *Space of Relations*, as shown in Appendix (Eq. (A2)),

$$\{\tilde{r}\}_s = e^{\{\tilde{\sigma} \circledR \tilde{i} \oplus \tilde{\varphi} \circledR \tilde{j} \oplus \tilde{\vartheta} \circledR \tilde{k}\}} \tag{24},$$

which represents one of the fundamental concepts underlying to the MOP, we can restructure Eq. (24) in a more explicit

form, because the values of $\tilde{\rho}_{1j}(t_0 + \Delta t), \tilde{\varphi}_{1j}(t_0 + \Delta t), \tilde{\theta}_{1j}(t_0 + \Delta t)$ are now contemporaneously known.

To this purpose, it is possible to preliminary restructure Eq. (24) in the form

$$\{\tilde{r}\}_s = \tilde{\rho} \circledR \tilde{i} \circledR e^{\{\tilde{\varphi} \circledR \tilde{j} \oplus \tilde{\vartheta} \circledR \tilde{k}\}} \tag{25}$$

and then, by considering the first order expansion series of the exponential, we can get the following Ordinal Expression of the translation pertaining to each Ordinal Uniance, considered in its three fundamental Ordinal components

$$\{\tilde{r}(t_0 + \Delta t)\}_{1j} = \tilde{\rho}_{1j}(t_0 + \Delta t) \circledR \tilde{i} \oplus \tilde{\rho}_{1j}(t_0 + \Delta t) \circledR \tilde{\varphi}_{1j}(t_0 + \Delta t) \tilde{j} \circledR \oplus \tilde{\rho}_{1j}(t_0 + \Delta t) \circledR \tilde{\theta}_{1j}(t_0 + \Delta t) \circledR \tilde{k} \tag{26}$$

where the development in series has been limited to the sole first order because of the substantially limited values of the angular variations of the each single Uniance.

Eq. (26), which is of Generative Ordinal Nature, can also be rewritten by expressing the variables $\tilde{\varphi}_{1j}$ and $\tilde{\theta}_{1j}$ in terms of their "associated cardinalities", that is in the form

$$\{r(t_0 + \Delta t)\}_{1j} = \tilde{\rho}_{1j}(t_0 + \Delta t) \tilde{i} \cdot \left(1 + \tilde{\varphi}_{1j}(t_0 + \Delta t) \tilde{j} + \theta_{1j}(t_0 + \Delta t) \tilde{k}\right) \tag{27}.$$

Eq. (27) then allows us to express the absolute value of any *total* Uniance, expressed in terms of the "associated cardinalities" of its three basic components

$$\left|\{\tilde{r}(t_0 + \Delta t)\}_{1j}\right| = \tilde{\rho}_{1j}(t_0 + \Delta t) \cdot \sqrt{1 + + \varphi_{1j}^2(t_0 + \Delta t) + \theta_{1j}^2(t_0 + \Delta t)} \tag{28}.$$

Such an expression, although formulated in terms of *"associated cardinalities"* of variables that are, in reality, of *Generative Nature*, allows us to better recognize the *valence* of the Ordinal concept of "Uniance", with respect to the concept of "distance", which is not a "generative" concept, because it is defined in an *absolute geometrical space* defined "a priori". This also means that the accelerated expansion of Galaxies previously considered should be better referred to the proper total value of the Uniances between the various Galaxies, more than to the sole "radial components" of the same Uniances". Consequently, by taking in account the previous levels of analysis, and by considering in particular the Exit of previous phase 3, the Accelerated Expansion of the Galaxies reveals as being an Ordinal Expansion that is *intimately pertaining* to the *Proper Space* of the same System, more than an Expansion that "evolves" in an absolute geometrical space defined "a priori".

## Conclusions

On the basis of the previous modalities of analysis, the Accelerated Expansion of any Galaxy Cluster can be seen as an "Emerging Ordinal Exit" of the same Astronomical System, when understood as a Self-Organizing System, thus analyzed in the light of the Maximum Ordinality Principle, and consequently in the absence of any specific reference of "Dark Energy" and "Dark Matter".

Such an "Emerging Ordinal Exit" may also suggest a *more general perspective*, always based on the MOP. In fact, it "reveals" that the same Accelerated Expansion of the *entire Universe* can be seen as a direct "Emerging Exit" of an *Ordinal Generativity*, which is specific of the same Universe *as Whole*, when it is understood as a Self-Organizing System.

A Generativity that diffuses its harmonious "effects" on all the various Galaxy Clusters of the Universe and, at the same time, on each single Galaxy included in them.

In this sense, such a General Perspective may also represent a valid presupposition to analyze, in a more general context, the Process of "Big Bang", always in the light of the Maximum Ordinality Principle, and thus in the absence of "Dark Energy" and "Dark Matter".

The previous Ordinal Analysis in fact is not understood as being a sort of "*recompositio ad unum*", but it is exactly understood in the opposite sense.

In fact, from the *very beginning* of the Ordinal Analysis, the "Attention" is devoted to the "Emerging Quality" of any specific Self-Organizing System. Where such an "Emerging Quality", understood as an "Irreducible Excess", manifests *its presence* in the *Harmony Relationships* of the same System. While such *Harmony Relationships*, in turn, represent the "Exceeding" Ostension of the *specific Generativity of the System*, which, at the same time, is *Self-Organizing*, of *Ordinal Nature*, and understood *as a Whole*.

This also means that, according to the Ordinal Perspective previously adopted, the "Attention" of the Analysis is essentially orientated toward to the Ordinal Generative Aspects of the System, with respect to the more habitual description of a System in "functional" terms. So that all the Relationships considered in the Analysis are all of Ordinal Nature, and not simply of "functional" nature. This evidently, and in particular, reflects on the adoption of the previous concept of "Uniance", instead of that of "distance".

In such a perspective, the Exit so obtained is not the "result" of a mere "algebraic" analysis, not even of a simple "functional" nature. This is because all the Relationships adopted, in particular those based on the concept of *Relational Space* $\{\tilde{r}\}_s$ (see Eq. (A2) in Appendix), are all of *Generative* Nature. In fact, they are all obtained and expressed by means of the innovative concept of the "Incipient" Derivative.

Consequently, the *Translation* of each Galaxy does not reduce to a simple "sum" of "local geometrical aspects", specific and characteristic of each Galaxy, but it is referable to *all the System* understood as a Whole, according to which all the different modalities of Translation are faithfully described by its Harmony Relationships.

This also means that any System, when understood as a Whole, cannot be considered as being "divisible in parts", understood as being "distinct and separated", as it happens in the case in which they are considered as being related between them in "functional" terms. In other words, inside a System understood as a Whole, each single "variation" of one "part" is not, by itself, an "efficient cause" that can "influence" all the other ones. This is because it is exactly true the "opposite": any "variation" of a single Galaxy is, by itself, only a simple Ordinal "Indicator" of a *global variation* of the Whole. Where such a "comprehensive variation", in turn, "reflects" in Ordinal terms on the same considered Galaxy, precisely because the latter is "intimately" related to the Whole.

This also shows that any Ordinal Self-Organizing System does not "evolves" or "expands" in a "geometrical space", defined "a priori" and of "absolute nature". But, in adherence to the same concept of *Relational Space*, each Ordinal System, on the basis of its same specific Generativity, "expands" its *Proper Relational Space* according to a Process in which the same System, as already shown in **[7]**, contextually "evolves" in *its Proper Time*. And this may also represent a valid

presupposition to analyze the Process of "Big Bang" (previously recalled) in a more general context, always in the light of the Maximum Ordinality Principle, in the absence of "Dark Energy" and "Dark Matter".

## Appendix. General Explicit Solution to the two Fundamental Equations of the MOP, understood as a Whole

The first Fundamental Equation (1) always presents an *explicit solution* [7], which can always be written in the general form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\left\{\begin{pmatrix}\tilde{\alpha}_{11}(t)\\\tilde{\alpha}_{21}(t)\\\cdots\\\tilde{\alpha}_{N1}(t)\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_{12}(t)\\\tilde{\alpha}_{22}(t)\\\cdots\\\tilde{\alpha}_{N2}(t)\end{pmatrix}\begin{pmatrix}\cdots\\\cdots\\\cdots\\\cdots\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_{1N}(t)\\\tilde{\alpha}_{2N}(t)\\\cdots\\\tilde{\alpha}_{NN}(t)\end{pmatrix}\right\}} \tag{A1}$$

where the *Relational Space* $\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}}$ depends on the Nature of the System analyzed, while the corresponding structure

of each term of the Ordinal Matrix depends on the *Specific Generativity* of the System $(\tilde{d}/\tilde{d}t)_s$.

For example, if the *Relational Space* of the System is represented by $\{\tilde{\rho}, \tilde{\varphi}, \tilde{\vartheta}\}$, that is by means three Ordinal topological coordinates always considered as *the exit of a Generative Process*, we can assume

$$\{\tilde{r}\}_s = e^{\tilde{\alpha}(t)} = e^{\{\sigma \circledR \tilde{i} \oplus \varphi \circledR \tilde{j} \oplus \vartheta \circledR \tilde{k}\}} \tag{A2},$$

where the adoption of the curly brackets "{}" is explicitly finalized to remind us that the Ordinal Structure is understood as a *Whole*.

This is because, on the basis of a generalized form of De Moivre representation, it is always possible to write

$$\{\tilde{r}\}_s = \{\tilde{\rho} \circledR \tilde{i} \circledR e^{\tilde{\varphi} \circledR \tilde{j}} \circledR e^{\tilde{\vartheta} \circledR \tilde{k}}\} = \{e^{\sigma \circledR \tilde{i}} \circledR e^{\tilde{\varphi} \circledR \tilde{j}} \circledR e^{\tilde{\vartheta} \circledR \tilde{k}}\} =$$

$$= e^{\{\sigma \circledR \tilde{i} \oplus \varphi \circledR \tilde{j} \oplus \vartheta \circledR \tilde{k}\}} = e^{\tilde{\alpha}(t)} \tag{A3},$$

where the traditional versors $\vec{i}, \vec{j}, \vec{k}$ are now replaced by three unit *spinors* $\tilde{i}, \tilde{j}, \tilde{k}$, which are defined in such a way as to satisfy the following *Relational Product* Rules:

$$\tilde{i} \circledR \tilde{i} = \oplus 1 \qquad \tilde{i} \circledR \tilde{j} = \tilde{j} \qquad \tilde{i} \circledR \tilde{k} = \tilde{k} \tag{A4.1}$$

$$\tilde{j} \circledR \tilde{i} = \tilde{j} \qquad \tilde{j} \circledR \tilde{j} = \ominus 1 \qquad \tilde{j} \circledR \tilde{k} = \tilde{k} \tag{A4.2}$$

$$\tilde{k} \circledR \tilde{i} = \tilde{k} \qquad \tilde{k} \circledR \tilde{j} = \tilde{k} \qquad \tilde{k} \circledR \tilde{k} = \ominus 1 \tag{A4.3}$$

where the symbols $\oplus$ and $\circledR$ express more intimate relationships between the same: both in terms of sum ($\oplus$) and in

terms of (relational) product ($\circledR$) with respect to the case of traditional versors $\vec{i}, \vec{j}, \vec{k}$.

So that representation (A3) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized

by one real unit $(\tilde{i})$ and two imaginary units ($\tilde{j}$ and $\tilde{k}$).

Equation (A1) thus describes the *Generative Evolution* of the System as the exit of an *Ordinal Cooperation* of *N* Co-Productions (vertical brackets) and their associated *N Inter-Actions* (horizontal brackets). At the same time, when the

Process has reached its Maximum Ordinality, each term $\tilde{\alpha}_{ij}(t)$ of the Ordinal Matrix, as a consequence of such a Maximization Process, will transform into a Binary-Duet Relationship, represented as $\{\tilde{\alpha}_{ij}(t)\}^{\{\tilde{2}/\tilde{2}\}}$.

At the same time, the adoption of the brackets "$\{\}$" in Eq. (A1) is explicitly finalized to remind us that the Ordinal Matrix represents the Ordinal Structure of the System understood as a *Whole*.

In fact, all the elements of the Ordinal Matrix (in Eq. (A1)) satisfy the following "Ordinal Relationships"

$$\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \overset{*}{=} (\sqrt[N-1]{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \quad (A5),$$

$$\text{for } j=1,2,3,\ldots N-1$$

where the additional terms $\{\tilde{\lambda}_{i,j}(t)\}^{\{\tilde{2}/\tilde{2}\}}$ explicitly account for the inter-relations between the correlative couples, together with their associated habitat conditions.

Eqs. (A5) can also be termed as "*Harmony Relationships*" precisely because they show that all the elements $\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$ of the Ordinal Matrix can be obtained by means of *one sole* arbitrary couple $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, assumed as reference, and the N-1 Ordinal Roots $(\sqrt[N-1]{\{\tilde{1}\}})_j$ of Unity $\{\tilde{1}\}$.

Consequently, if each element of the Ordinal Matrix (in Eq. (A1)) is expressed in terms of the reference couple $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$, the solution to the first Fundamental Equation (1) assumes the following form

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ\left\{ \begin{bmatrix} (\sqrt[N-1]{\{\tilde{1}\}})_{11} \\ (\sqrt[N-1]{\{\tilde{1}\}})_{21} \\ \cdots \\ (\sqrt[N-1]{\{\tilde{1}\}})_{N1} \end{bmatrix}, \begin{bmatrix} (\sqrt[N-1]{\{\tilde{1}\}})_{12} \\ (\sqrt[N-1]{\{\tilde{1}\}})_{22} \\ \cdots \\ (\sqrt[N-1]{\{\tilde{1}\}})_{N2} \end{bmatrix} \cdots \begin{bmatrix} (\sqrt[N-1]{\{\tilde{1}\}})_{1N} \\ (\sqrt[N-1]{\{\tilde{1}\}})_{2N} \\ \cdots \\ (\sqrt[N-1]{\{\tilde{1}\}})_{NN} \end{bmatrix} \right\}} \quad (A6)$$

where, for the sake of simplicity, the term $\{\tilde{\alpha}_{12}(t) \oplus \tilde{\lambda}_{12}(t)\}$ stands for $\{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\}$.

The same Ordinal Matrix, in addition, may always be represented in a more synthetic form by means of one sole symbol, by adopting the following synthetic notation

$$\{(\sqrt[N-1]{\{\tilde{1}\}})_{ij}\}^{\uparrow\{\tilde{N}/\tilde{N}\}} \quad (A7),$$

where the arrow "↑" explicitly reminds us that the Ordinality $\{\tilde{N}/\tilde{N}\}$ has always to be considered as being a particular form of *Over-Ordinality*.

In this way the explicit solution to the first Fundamental Eq. (1) can synthetically be expressed as follows

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{\alpha}_{12}(t)\oplus\tilde{\lambda}_{12}(t)\}\circ(\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \quad (A8).$$

Consequently, if such a solution is introduced into the Global Feed-Back Process represented by the second Fundamental Equation (2), the latter transforms into a typical Riccati's Equation of *Ordinal Nature*, whose explicit solution is given by

$$\{\tilde{r}\} = e^{\{\tilde{\alpha}(t)\}} = e^{\{\tilde{B}(t)\}\circ(\sqrt[N-1]{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}} \quad (A9),$$

where

$$\tilde{B}(t) = \left\{ \begin{pmatrix} \oplus\,\tilde{A}(t) \\ \Theta\,\tilde{A}(t) \end{pmatrix}, \begin{pmatrix} \Theta\,\tilde{A}(t) \\ \oplus\,\tilde{A}(t) \end{pmatrix} \right\} \tag{A10}$$

and

$$\tilde{A}(t) = \{\{\tilde{\alpha}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(0)\}^{\{\tilde{2}/\tilde{2}\}}\} \circ ({}^{N-1}\!\!\sqrt{\{\tilde{1}\}})^{\uparrow\{\tilde{N}/\tilde{N}\}}\}^{(\tilde{2}/\tilde{2})} \oplus \oplus \ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\}) \tag{A11},$$

in which the term $\ln(\tilde{c}_1 \oplus \{\tilde{c}_2, t\})$ accounts for the *origin and habitat conditions* of the Feed-Back Equation and, at the same time, also represents an *Over-Ordinality* contribution specifically due to the same Feed-Back Process. A contribution which is particularly important for *the System stability* when the System interacts with a System of its Habitat.

Equation (A9) then represents the Explicit "Emerging Solution" to the Maximum Ordinality Principle, formulated in two "Incipient" Differential Equations ((1) and (2)), considered as being a Whole.

## A1. Explicit Solution (A8) reformulated in operative terms with reference to a given time $t_0$

In order to get an explicit solution which may result much easier to program on a computer and, in particular, on a PC, the Solution (A8) can be restructured in more operative terms as follows

a) $\qquad \tilde{\rho}_{1j}(t_0) = \tilde{A}\cdot e^{\tilde{S}_l(t_0)}$ (A12) where $\qquad \tilde{S}_l(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Sigma}_0 - C_l \cdot (\tilde{\Phi}_0 + \tilde{\Theta}_0)]$ (A12.1)

b) $$\tilde{\varphi}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Phi}_0 - C_l \cdot \tilde{\Sigma}_0] \tag{A13}$$

c) $$\tilde{\theta}_{1j}(t_0) = \psi_1 \cdot E_l \cdot [B_l \cdot \tilde{\Theta}_0 - C_l \cdot \tilde{\Sigma}_0 + C_l(\tilde{\Phi}_0 - \tilde{\Theta}_0)] \tag{A14}$$

where $\qquad B_l = \cos(\sqrt{2}\cdot\psi_l) \qquad C_l = D_l = \dfrac{1}{\sqrt{2}}\sin(\sqrt{2}\cdot\psi_l)$ (A15)

and $$\psi_l = \psi_2 \cdot \frac{\varepsilon_2 + 2\pi\cdot l}{N-1} \tag{A16}.$$

In the previous Relationships:

i) $\tilde{\Sigma}_0, \tilde{\Phi}_0, \tilde{\Theta}_0$ represent the Ordinal coordinates of the reference couple, generally termed as "couple 12", considered at the time $t_0$. Then the symbols $\Sigma_0, \Phi_0, \Theta_0$ synthetically stand for $\{\tilde{\sigma}_{12}(t_0), \tilde{\varphi}_{12}(t_0), \tilde{\vartheta}_{12}(t_0)\}$

ii) the Ordinal factor $\psi_1 \cdot E_l$ originates from the assumption that, in Eq. (A5), which represent the Harmony Relationships, here reproduced for the sake of clearness

$$\{\tilde{\alpha}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{i,j+1}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \overset{*}{=} ({}^{N-1}\!\!\sqrt{\{\tilde{1}\}})_j \circledR \{\tilde{\alpha}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}} \oplus \{\tilde{\lambda}_{12}(t)\}^{\{\tilde{2}/\tilde{2}\}}\} \tag{A17},$$

$$\text{for } \ j=1,2,3,....\text{N-1}$$

the terms $\{\tilde{\alpha}_{1,j+1}(t)\}$, after a previous reduction of its Ordinality $\{\tilde{2}/\tilde{2}\} \to 1$, can be directly expressed in terms of a *specific periodicity* $E_l = \dfrac{\varepsilon_1 + 4\pi\cdot l}{N-1}$ (A18), which, at the same time, is modulated by the Ordinal factor $\psi_1$;

iii) Then, after having rewritten the Ordinal Relationships in the form

$$Exp\{\tilde{\sigma}_{1j}(t_0),\tilde{\varphi}_{1j}(t_0),\tilde{\vartheta}_{1j}(t_0)\}\overset{*}{=}Exp[(\sqrt[N-1]{\tilde{1}})_l\circledR\{\tilde{\sigma}_{12}(t_0),\tilde{\varphi}_{12}(t_0),\tilde{\vartheta}_{12}(t_0)\}] \qquad (A19)$$

iv) and after having assumed the explicit expression of the Ordinal Roots of Unity as follows

$$(\sqrt[N-1]{\tilde{1}})_l\overset{*}{=}Exp\{\tilde{\alpha}\circledR\tilde{i}\oplus\tilde{\beta}\circledR\tilde{j}\oplus\tilde{\gamma}\circledR\tilde{k}\} \qquad (A20),$$

where $\qquad \alpha=\dfrac{\varepsilon_1+4\pi\cdot l}{N-1} \qquad \beta=\dfrac{\varepsilon_2+2\pi\cdot l}{N-1} \qquad \gamma=\dfrac{\varepsilon_3+2\pi\cdot l}{N-1} \qquad (A21),$

the expansion series of Eq. (A19), together with the contextual adoption of the Rules of the Ordinal Product (A4.1), (A4.2), (A4.3), leads to the Ordinal Relationships (A12), (A12.1), (A13), (A14) initially introduced, with the associated coefficients expressed by Eqs. (A15), (A16).

For the sake of completeness it is worth adding that:

- The symbol $\{\tilde{1}\}$ represents *the Unity of the System* (understood as a Whole) by means the representation of the *Unity* of its *Proper Space of Relations*

- $\varepsilon_1,\varepsilon_2,\varepsilon_3$ characterize the spatial orientation of the System as a Whole, with reference to its Ordinal Proper Space

- in Eq. (3.7) the "periodicity" of the "spinor" $\tilde{i}$ is assumed equal to $4\pi$, because it is expressed in steradians

- while the periodicity of the spinors $\tilde{j}$ e $\tilde{k}$ are both equal to $2\pi$ radians, because these spinors are always "orthogonal", both among them and with respect to the spinor $\tilde{i}$. An "orthogonality" that can be seen as a form of reciprocal "irreducibility" (as also indicated by the same Relational Products);

- while the Factor "$\tilde{A}$" represents an *Internal Ordinal Factor* according to which all the *radial* Uniances of the various Couples are appropriately referred to the *radial* Uniance of the Reference Couple "12" and, at the same time, on the basis of the "associated" cardinality of the latter, they are all expressed in terms of a desired scale measure (e.g. in Mpc).

## A2. General Considerations on the Explicit Solution reformulated in operative terms

From the previous exposition, it should result as being evident that the Harmony Relationships represent an "Irreducible Excess". That is an "Exceeding" manifestation of the *Generativity of the System*, where the latter is at the same time *Self-Organizing*, of *Ordinal Nature*, and understood as *a Whole*.

This means that the Explicit Solution reformulated in *operative terms*, precisely because obtained from the Harmony Relationships, also represents a correlative "Emerging Solution". And even if the operative Solution is expressed in terms of the three "components" of *each single couple* "*1j*", such "components", although recognizable as being "distinct", they can never be thought as being conceptually "separable", because each triad of coordinates represents one sole entity of Ordinal Nature.

The same concept is even more valid and particularly important with reference to the coordinates of *all the couples* that form the System. In other terms, the coordinates furnished by the Operative Solution are not conceptually "separable" between them, *neither with reference to each single couple*, *nor* with reference to *all the various couples* of the System as a Whole.

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