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Toward a Mathematical Formulation of the Maximum Em-Power Principle

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Abstract. **This paper aims at presenting a possible logical *route* toward a mathematical formulation of the well-known "Maximum Em-Power Principle" (Lotka-Odum) in accordance with its several and equivalent general verbal definitions existing in literature.**
To this purpose the three following successive *steps* will be considered:
(i) a rigorous definition of Emergy in mathematical terms;
(ii) a general mathematical formulation of the Emergy Balance Equation (stated according to Emergetic Algebra and globally valid for Systems no matter how complex);
(iii) the fundamental logical presuppositions for a possible mathematical formulation of the "Maximum Em-Power Principle".
For the sake of generality, the three previous conceptual steps will be also analysed at three different hierarchical levels of time-dynamic system behaviour: steady state conditions, stationary conditions and generally variable conditions.
The paper will not (expressly) deal with the question as to whether the considered Principle is the so-called "Fourth Thermodynamic Principle" or not, but only with the possibility of establishing solid bases for a correct answer to this question (ever present as sub-jacent thought).
Nevertheless, the work done up to now (and here presented in a very synthetic form) should be already sufficient to understand that we are trying to mathematically express *something* which is (very likely) more than a *Thermodynamic* Principle.

1. Introduction

The "Maximum Em-Power Principle" (Lotka-Odum) is generally considered as the "Fourth Thermodynamic Principle" (mainly) because of its *practical validity* for a very wide class of physical and biological systems, although its general formulation is not yet rigorously defined in mathematical terms. Under these conditions it is very difficult to answer the following fundamental questions:
Is the "Maximum Em-Power Principle" a *Thermodynamic* Principle ? Is it really *independent* from other well-known Principles (like Energy conservation and Exergy degradation)?
In order to give a contribution to a definitive answer to these questions we have begun analysing a possible resolutive *route* that should lead to a fundamental result: a general mathematical formulation of the considered Principle.
The logical way we have hypothesised may be therefore articulated in the following three successive:
(i) a rigorous definition of Emergy in mathematical terms
(ii) the statement of a pertinent Emergy Balance Equation (according to Emergetic Algebra)
(iii) the fundamental logical presuppositions for a possible mathematical formulation of the "Maximum Em-Power Principle".
Each of the previous steps will be also analysed, for the sake of generality, at three different hierarchical levels of time-dynamic system behaviour: *steady state* conditions, *stationary* conditions and *generally variable* conditions.

2. Mathematical definition of Emergy

The verbal definition of Emergy given by Prof. H. T. Odum (1984) is, in reality, a very general definition. It may be considered valid under completely variable conditions too. The reference to solar available Energy is only a *practical* way in order to calculate its value. Its definition, in fact, may be based on whatever other form of reference available Energy.

In spite of its wide and recognised generality, Emergy is evaluated (in practical and usual applications) preferably with reference to *steady state conditions* (especially for those natural systems that have reached their optimum "working point" as a consequence of natural selective processes) and these values are also the basis for Transformity calculations. In addition, when Systems are analysed under more general conditions (e.g., stationary conditions), the previous Transformities are used without substantial modifications and Emergy variations are assumed to be mainly due to time variations of some associated variable quantities (e.g., mass, Energy, etc.). This usual procedure however does not exclude that, in much more general studies, Transformities might also be thought of as functions of time.

Taking into account the basic perspective assumed in the development of this work, it is clear that an Emergy definition should be the most general possible. We re-propose a completely general mathematical definition of Emergy (Ref. [7],[8]) that will be now analysed in more detail.

From a mathematical point of view, the general verbal definition of Emergy given by Prof. Odum as «the total *solar* equivalent *available Energy* directly and indirectly *used up* to generate a specific form of Energy (or product)» may be expressed in terms of Exergy through *the specific rules of Emergetic Algebra* (see par. 3) which reflect the fact that Emergy is not a conservative quantity.

In fact we may define a quantity $Em^*(t)$ as follows

$$Em^*(t) = \int_{-\infty}^{t} \dot{E}x_{eq}(\tau)d\tau \qquad (2.1)$$

where $Ex_{eq}(\tau)$ is defined as

$$Ex_{eq}(\tau) = \int_{D^*(\tau)} c(x,y,z,\tau)\cdot\rho(x,y,z,\tau)\cdot ex(x,y,z,\tau)d_3V \qquad (2.2)$$

and $\dot{E}x_{eq}(\tau)$ is the instantaneous *equivalent* Exergy Power used up during the process of generating a specific product.

In Eq. (2.2) c is a *dimensional structural factor* (whose dimensions are *sej/J*, that is solar emergy joules per joule) which depends, among other things, on *co-injection* or *co-production factors* (as we will see later) and it is defined in such a way as *to summarise all the rules of the Emergetic Algebra* (this is the reason for the term "equivalent"); $D^*(\tau)$ is the Domain of integration which defines the quantity of the considered matter, $\rho(x,y,z,\tau)$ is the mass density, $ex(x,y,z,\tau)$ is the specific Exergy, while Newton's "dot" notation in Eq. (2.1) stands for the *total derivative* with respect to time.

It is then easy to recognise that, when we assume a *Lagrangian* perspective and steady state conditions, Eq. (2.1) defines the traditional and usual concept of Emergy, while *under absolutely variable conditions* Eq. (2.1) defines Emergy in its *widest* and *most general conception*.

2.1 Detailed analysis of the proposed mathematical definition of Emergy

1) *"Solid Circle" Definition*: Equation (2.1) depends on the *structural factor* $c(x,y,z,\tau)$ which, in turn, is strictly related to the concept of Transformity, and thus it is also dependent on the Emergy Balance Equation. Therefore its rigorous meaning will become completely clear after having shown its relationship with Transformity and having stated a general formulation of the Emergy Balance Equation.

2) *Basic reference to Exergy*: the definition of Emergy in terms of Exergy is preferable because of the following fundamental reasons:
(i) the expression "*available energy*" (especially in Anglo-Saxon literature) generally stands for "Exergy"; although these two concepts are practically interchangeable in the case of very simple systems, they are not really equivalent for those very complex ones: total Exergy in fact is *always* an additive function of the corresponding sub-system quantities, whereas *available energy* generally is not (Ref. [6]);
(ii) reference to Exergy also constitutes a very solid thermodynamic *starting point* for Emergy definition, although this fact *should not lead to thinking* that Emergy might be (*in any case*) reduced to Exergy.
In fact, *specific rules of Emergy accountability* constitute what *really* and *actually transforms* the considered exergetic contributions into a *wholly new and different quantity*. So that the difference in *dimensional units* is not *simply* a conventional way of expressing "identical" contributions (just considered differently), but they are a consequential expression of a new "dimension" according to which "we are looking at things".
3) *Lagrangian perspective*: it seems to be the most adequate perspective, at least as far as the *definition* of Emergy is concerned. In fact it corresponds to the usual "perspective" of evaluating (according to Emergetic Algebra) all the exergetic contributions spent (during the past, from a pre-defined *time origin*) in order to generate a "given system" at present under consideration.
This perspective, especially and properly indicated *for Emergy definition,* does not exclude an alternative (and equivalent) Eulerian description which, vice-versa, seems to be more suitable for formulating an Emergy Balance Equation (as we will see afterwards).
4) *Time origin definition*: time origin may be chosen in an arbitrary way: Eq. (2.1), in fact, is written under the only hypothesis that the time t (upper extreme of the integral) corresponds to the *present* time.
Moreover, as far as the presence of the quantity $-\infty$ is concerned, it is evident that this *mathematical lower extreme* enables us to choose the best indicated *time extreme* according to the specific system under consideration. In fact in our analyses the lower *time extreme* cannot be less than the origin of the Universe. In addition, for usual analyses, a *time extreme* not lower than the origin of Earth is surely more appropriate. In any case it is sufficient to assume all the concerned quantities to be equal to zero when the variable of integration is less than the prefixed lower extreme.
5) *Relative Emergy values*: what has just been recalled may be also expressed in an alternative way which will prove particularly useful in the analysis of the next point as well.
In fact Eq (2.1) may be re-written as follows

$$Em^{*}(t) = Em_0 + \int_{t_0}^{t} \dot{E}x_{eq}(\tau)d\tau \qquad (2.3),$$

so that what we previously said is equivalent to neglecting Em_0, and this corresponds to the fact of considering only *relative values* of Emergy. On the other hand we are not interested in *absolute* Emergy values, but only in *definite* Emergy *variations* (like we usually do for Energy too).
6) *Reference to solar Emergy*: this reference, although not obligatory, suggests some other interesting considerations concerning the *relative value* of Emergy assumed as reference. In fact, if we calculate Emergy associated with *solar Energy* by assuming as lower time extreme the time corresponding to the origin of the Sun ($6 \div 7$ billion years ago) and as reference *available energy* (= exergy) the one associated with cosmic radiation, we may find, according to definition (2.1), a huge value that, on the other hand, is not of *practical* interest for our *usual* Emergy Analyses. It is then common practice to (conventionally) *assume* Solar Emergy as (numerically) "equivalent" to its corresponding available energy (= exergy) and this, in turn, is equivalent to assuming solar Transformity equal to 1 *sej/J*.

This practical way of operating (that may be extended to whatever form of available Energy assumed as reference) implies a *fundamental aspect* that has to be clearly pointed out: any conventional assumption, as far as the Exergy of reference is concerned, is *generally and rigorously* valid *only if* we demonstrate that Emergy is a *state variable* (at least in the most general sense of Systems Theory).

7) *Total derivative*: the geometric surface of the ideal control Domain $D^*(\tau)$ (that delimits the System under consideration) is generally in movement in our reference system (Lagrangian perspective), while flows of different quantities (mass, Energy, Exergy, etc.) pass though the frontier (in both directions), that is they are characterised by a *relative velocity* with respect to the domain frontier.

It is then clear that *total derivative* (see expression (2.2)) is the most appropriate derivation technique to account for both variations: *inside* the control Domain and *through* its frontier. At this stage there are several possibilities of expressing these terms according to the specific aspects under consideration. In this theoretical context we have chosen the most general expression for a control Domain in movement with an absolutely arbitrary distribution of velocity in its internal parts, and especially on its frontier. Thus, by remembering the Leibnitz Theorem, we may write

$$\frac{d}{dt}Ex_{eq}(t) = \frac{d}{dt}\int_{D^*(\tau)}(c\cdot\rho\cdot ex)\cdot d_3V = \frac{\partial}{\partial t}\int_{D^*(\tau)}(c\cdot\rho\cdot ex)\cdot d_3V + \int_{\partial D^*(\tau)}(c\cdot\rho\cdot ex)\cdot v_{sn}\cdot d_2S \qquad (2.4)$$

where $v_{sn} = \vec{v}_s(x,y,z,\tau)\cdot\vec{n}$ is the distribution of system velocities normal to the frontier $\left[\partial D^*(\tau)\right]$.

2.2 Direct relationship between the Structural Factor $c(x,y,z,\tau)$ and Transformity

The Emergy value at the time t given by Eq. (2.1) may also be expressed as an integral of volume (extended to the Domain $D^*(t)$) through the pertinent values of the "local" properties [$\rho(x,y,z,t)$ and $ex(x,y,z,t)$] *at the same time* t and through a *Comprehensive structural factor* $C(x,y,z,t)$ considered as a function of the *same* "*local*" *time-space co-ordinates*:

$$Em(t) = \int_{D^*(t)} C(x,y,z,t)\cdot\rho(x,y,z,t)\cdot ex(x,y,z,t)d_3V \qquad (2.5).$$

If we now compare the obtained expression with the one that gives the same Emergy value through the concept of Transformity

$$Em(t) = \int_{D^*(t)} C(x,y,z,t)\cdot\rho(x,y,z,t)\cdot ex(x,y,z,t)d_3V = Tr(t)\int_{D^*(t)}\rho(x,y,z,t)\cdot ex(x,y,z,t)d_3V \qquad (2.6)$$

we can draw some interesting conclusions:

(i) Transformity is a *mean value* (referred to the Domain $D^*(t)$) that directly relates total *Emergy* (at the time t) to the corresponding content of *available Energy* (or *Exergy*) in the system (at the same time t);

(ii) its concept (and its corresponding value) is directly associated to *structural* "properties" that show the *progressive increase of information content* acquired by the system during all the productive process (see Eq. (2.1) and especially the *structural factor* $c(x,y,z,\tau)$ in Eq. (2.2));

(iii) this increased content of information is the "Quality" that, even though associated to a limited amount of available Energy, "qualifies" this Energy as far as its *relational capacity* is concerned, that is its capacity of doing "*work*" (which, according to Prof. Odum, cannot be reduced to the mere *mechanical* work);

(iv) Eq. (2.6) also contributes to (even more clearly) pointing out the profound difference between the Emergy concept and more traditional Energetic (or Exergetic) concepts;

(v) Moreover Eq. (2.6) shows that there is also an *alternative possibility of determining Transformity*, that is not only through an «external» procedure (the traditional one), but also through the direct («internal») determination of the *global content of information* which is present in the system at the considered time.
As far as the this last aspect is concerned, we may observe that this evaluation procedure is not very often followed by various Scientists (probably because it is a little more difficult), although Prof. Odum indicates how to relate both quantities through the general concepts of *Information Theory* (Ref.[4]).

3. Emergy Balance Equation (in steady state and variable conditions)
An *Emergy Balance Equation* that might be defined as *completely general* (that is applicable under whatever conditions, as in the case of the Energy Balance Equation) has not been formulated yet.
There are some Emergy Accounting Procedures that constitute the so-called *Emergetic Algebra*. Before dealing with a possible general balance equation it is worth recalling the fundamental rules that have to be taken into account in a generalisation process, by also remembering that these rules *generally* refer to steady state conditions (Ref. [3]):

1st rule: "*All Source Emergy to a Process is assigned to the Process's output*"
2nd rule: "*By-products from a Process have the total Emergy assigned to each pathway*"
3rd rule "*When a pathway splits, the Emergy is assigned to each "leg" of the split based on their percent of the total Energy flow on the pathway*"
4th rule "*Emergy cannot be counted twice within a system. In particular*:
a) Emergy in feedbacks should not be double counted
b) by-products, when reunited cannot be summed".

In the perspective of formulating an absolutely general mathematical Emergy Balance Equation, it is really fundamental to distinguish between Emergy *sub-System Balances* and *Global System Balance.*

3.1 Emergy Sub-System Balances
If we consider a very simple sub-System like a «co-production process» in steady state conditions (Fig.1,a), according to the above mentioned rules we may write in Emergy units, but also in Emergetic flows)

$$Em(u) = \frac{1}{2} Em(y_1) + \frac{1}{2} Em(y_2) + 0 \cdot Em(y_3) \qquad (3.1) \qquad \text{with} \qquad Em(y_1) = Em(y_2) \qquad (3.2).$$

The two coefficients $\frac{1}{2}$ may be defined as «co-production coefficients» in the sense that they properly express the fact that each considered «output» is actually «one out of the two» by-products, whereas the «zero» coefficient expresses that there is «no co-productive correlation» with the other two (o more) by-products.
In general, in the case of more than 2 by-products, we may write (by neglecting uninfluential terms)

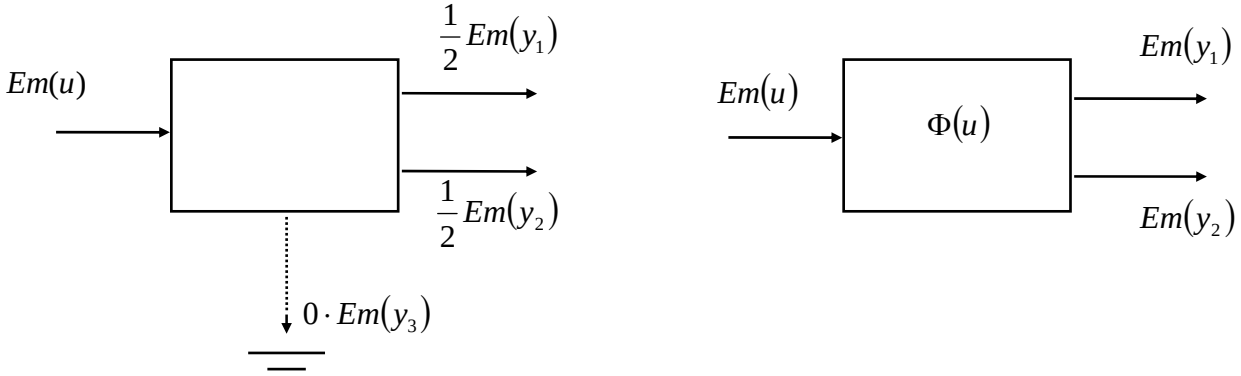
$$Em(u) = \sum_{j=1}^{n} \beta_j \cdot Em(y_j) \qquad (3.3) \qquad \text{where} \qquad \beta_1 = \beta_2 = \ldots = \beta_n = \frac{1}{n} \qquad (3.4).$$

The same considerations may be applied to *input* quantities too. In fact input quantities are *not always* Emergetically independent (like solar Emergy and geothermal Emergy). Thus we may analogously consider the Emergetic "sum" of such inputs through similar *co-injection* coefficients (α_j) in order to synthetically write

$$\sum_{j}^{m} \alpha_j \cdot Em(u_j) = \sum_{l}^{n} \beta_l \cdot Em(y_l) = \frac{1}{n} \sum_{l}^{n} Em(y_i) \qquad (3.5).$$

On the other hand, if inputs are Emergetically independent, their co-injection coefficients are equal to 1 (otherwise there is a certain *co-dependence* in their «in-jection» of Emergy *into* the considered sub-System).

1st Equivalent mathematical model **2nd Equivalent mathematical model**



Figs. 1,a & 1,b. MATHEMATICAL MODELS FOR A CO-PRODUCTION PROCESS

In order to get a *general formulation* that might be valid (in perspective) under completely variable conditions too, it is useful to "translate" in a "mathematically equivalent form" the Emergetic rules previously presented. Eq. (3.1) may be, for instance, re-written as follows

$$Em(u) + \Phi(u) = Em(y_1) + Em(y_2) \qquad (3.6)$$

where $\Phi(u)$ is an Emergy flow «Source Term» (depending on input quantities, see Fig. 1,b) which accounts for the exact amount that makes Eq. (3.6) algebraically equivalent to Eq. (3.1), that is

$$\Phi(u) = Em(u) \qquad (3.7).$$

In the case of n by-products $(n \geq 2)$, the Source Term is then given by the expression

$$\Phi(u) = (n-1)Em(u) \qquad (3.8).$$

Such a procedure may be analogously followed *for whatever elementary sub-System*.
Let us consider, as a further example, an «interaction process» (Fig. 2,a).
According to the first rule of Emergetic Algebra we may write (in steady state conditions)

$$Em(u_1) + Em(u_2) = Em(y) \qquad (3.9).$$

Output Emergy may be also written in a form of product

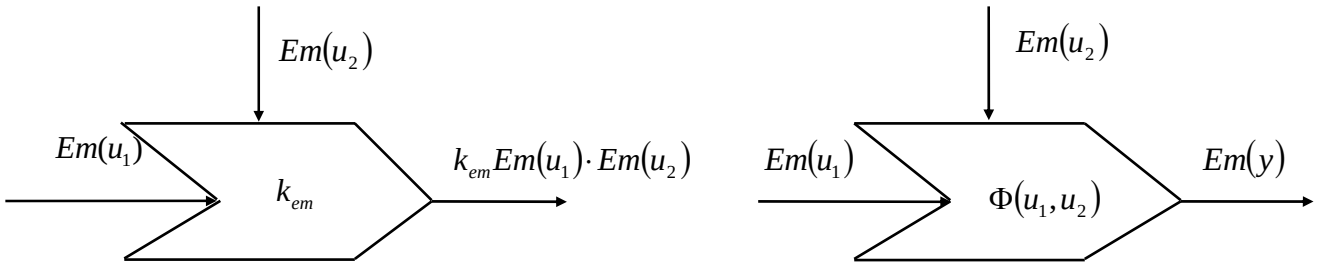
$$Em(y) = k_{em} \cdot Em(u_1) \cdot Em(u_2) \qquad (3.10)$$

where k_{em} is given by $$k_{em} = \frac{Em(u_1) + Em(u_2)}{Em(u_1) \cdot Em(u_2)} \qquad (3.11).$$

At this stage Eq. (3.9) may be easily re-structured in the same form as Eq. (3.6) by introducing a «Source Term» $\Phi(u_1, u_2)$ so that, in steady state conditions, it might give the same quantitative balance (see also Fig. 2,b)

$$Em(u_1) + Em(u_2) + \Phi(u_1, u_2) = Em(y) \qquad (3.12).$$

Basic mathematical model **Equivalent mathematical model**



Figs. 2,a & 2,b. MATHEMATICAL MODELS FOR AN INTERACTION PROCESS

But $\Phi(u_1, u_2)$ may assume, *in principle*, a more general structure; like (e.g.) the following one

$$\Phi(u_1, u_2) = \left[Em(u_1) + Em(u_2)\right] \cdot \left\{ \frac{k^*_{em} \cdot Em(u_1) \cdot Em(u_2)}{Em(u_1) + Em(u_2)} - 1 \right\} \qquad (3.14).$$

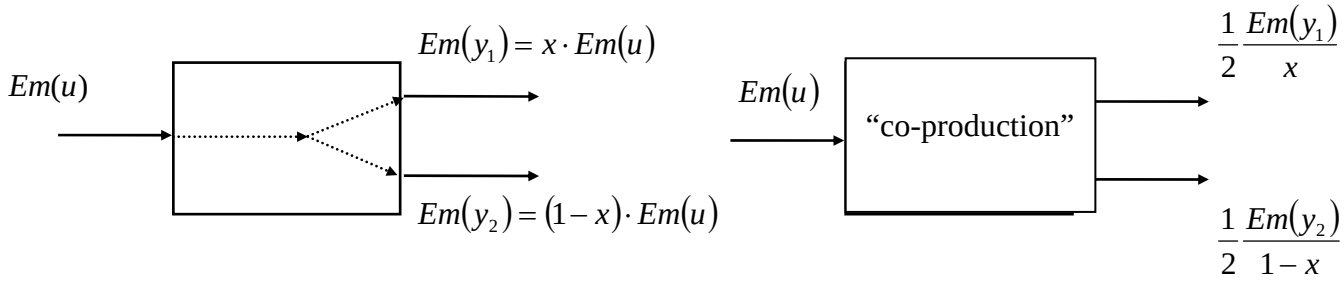
It is then clear that, if we are analysing an «interaction process» under steady state conditions, we will certainly assume

$$k^*_{em} = k_{em} \qquad (3.15);$$

but the Source Term, under generally variable conditions, could also be different from the one considered in steady state conditions. This is the reason for having assumed a more general structure.

Another simple sub-System that may sometime be very useful to represent in a mathematically equivalent way (as we will see later on) is the «split process» (Fig.3,a). This one, in fact, may be modelled as an "equivalent" «co-production process» (Fig.3,b).

Basic mathematical model **Equivalent mathematical model**



Figs. 3,a & 3,b. MATHEMATICAL MODELS FOR A SPLIT PROCESS

To this purpose it is sufficient to write the following equivalent balance

$$Em(u) = \frac{1}{2} \frac{Em(y_1)}{x} + \frac{1}{2} \frac{Em(y_2)}{1-x} \qquad (3.16) \qquad \text{with} \qquad \frac{Em(y_1)}{x} = \frac{Em(y_2)}{1-x} \qquad (3.17)$$

where x and $(1-x)$ are the pertinent percent of the total Energy flow. Moreover, in the case of particular necessity, a «split process» may also be mathematically represented through an *equivalent Source Term*, in the same way as already shown for a «co-production process».
As a conclusion of this paragraph we can say that this *mathematically equivalent procedure* based on the possible consideration of *particular* «Source Terms» has the following specific advantages:
(i) it is a procedure perfectly adherent to the rules of Emergetic Algebra
(ii) it may be applied to the analysis of whatever sub-System
(iii) it is *already perspectively orientated* at formulating a General Emergy Balance in a *structural analogy* with other Thermodynamic Balances.

3.2 Emergy Balance Equation for Complex Systems
Given a complex System made up of n elementary sub-Systems, we may say that, in principle, the Whole System is completely described (from an Emergetic point of view) by n *linear* equations (like Eqs. (3.6) or (3.12)) corresponding to the n considered sub-Systems. However this mathematical representation *does not allow* us to point out all the *peculiarities* of Emergy Analysis. A more significative mathematical representation may be obtained by carrying out a *Global Emergy Balance* (for the Whole System) structured in such a way as to point out «*total equivalent* input/output Emergy contributions» to the System. Therefore, through an *adequate* linear combination of the n linear equations describing the corresponding n sub-Systems, we may always write (by taking especially into account the 4th rule of Emergetic Algebra) the following Global Emergetic Balance Equation (*in steady state conditions*)

$$\sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot Em(u_j) + \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot Em(y_l) \qquad (3.18)$$

where

α_j, β_l are the *co-injection* and *co-production* coefficients for each sub-System

α_j^*, β_l^* are their associated *re-normalisation factors* (referred to the Whole System)

$\Phi_k^*(u_1, u_2, .., u_m)$ is the "*equivalent*" Source Term relative to the *k-th* sub-System

γ_k, γ_k^* are the corresponding "weight" and its associated *re-normalisation factor*.

Some more details about the quantities present in Eq. (3.18) will be given afterwards with reference to a more general equation carried out in the next paragraph.

3.3 Emergy Balance Equation in variable conditions
As previously mentioned, the introduction of «Source Terms» is also (and mainly) useful in the perspective of formulating a General Balance Equation valid under *conditions however variable*. Firstly, because *each* Source is now an *effective dynamic term*, generally different from zero (whereas in steady state conditions it may be just an "equivalent" static term). Secondly, because each specific contribution (previously depending only on input quantities) may be now higher than the one due to a perfect input-output sub-System balance. Thirdly, because the *presence of a Source Term is essential* in order to give a *general structure* to each balance equation. In fact each sub-System equation may in this way be easily *completed* by simply adding a pertinent and specific *Accumulation Term.*
Let us consider Eq. (3.1) as an example of possible generalisation procedure of the Balance Equation. Eq. (3.1) now evidently becomes (through Eq. (3.7) and in terms of Eulerian description)

$$\dot{E}\, m(u) + \dot{\Phi}(u) = 2 \cdot \frac{\partial}{\partial t} A_D(t) + 2 \cdot \left[\frac{1}{2} \dot{E}\, m(y_1) \right] + 2 \cdot \left[\frac{1}{2} \dot{E}\, m(y_2) \right] \qquad (3.19)$$

where the "*dot*" stands for *Emergy flow* (or Em-power) and the *Accumulation Term* $A_D(t)$ is given by

$$A_D(t) = \int_{D(t)} C(x,y,z,t)\cdot \rho(x,y,z,t)\cdot ex(x,y,z,t) d_3V \qquad (3.20).$$

If we now follow the same procedure (previously described in steady state conditions) in order to carry out a *unique* Global Balance Equation (for the Whole System) we may consequently write

$$\sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}\, m(u_j) + \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \dot{\Phi}_k^*(u_1,u_2,..,u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}\, m(y_l) \quad (3.21)$$

where the only "structural novelty", with respect to Eq. (3.18), is represented by the *Global Accumulation Term* $A_{D_s}(t)$. This term (as will prove clearer through a detailed analysis of a specific example) does not reduce to a simple "sum" of the n sub-System accumulation terms, but is given by an *appropriate sum* of n "*equivalent*" accumulation terms, each one (in turn) expressed as a linear combination of all the n *real* sub-System contributions. That is, in formulae:

$$A_{D_s(t)} = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot A_{D_i}^*\left(A_{D_1}, A_{D_2}, .., A_{D_n}\right) = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot \sum_{j=1}^{n} \varepsilon_{ij}^* \cdot A_{D_j} \qquad (3.22).$$

The deepest meaning of this term, which is worthy of particular attention, will be illustrated through an application of the Global Balance Equation to a sufficiently Complex System (Fig. 4)(Ref. [3]). This practical application will also illustrate the meaning of some other terms that appear in equation (3.21) and contemporarily to show, through a detailed analysis of the considered System, the wide and comprehensive potentialities of the stated *Global Balance Equation* (3.21).

Let us then consider the System shown in Fig. 4 under some successive different conditions:

A) *Steady state conditions*, without *equivalent* Source Terms [in D and E].

Under these conditions Eq. (3.21) becomes

$$1\cdot \dot{E}\, m(S) + 1\cdot \dot{E}\, m(F) = \frac{1}{2}\dot{E}\, m(Z) + \frac{1}{2}\left[4\cdot \dot{E}\, m(Y)\right] \quad (3.23) \quad \text{with} \quad \dot{E}\, m(Z) = \left[4\cdot \dot{E}\, m(Y)\right] \quad (3.24).$$

As we can see, there is no necessity of particular input co-injection factors because there are only two *independent* inputs (at the level of the Whole System), whereas one of the two *by-products* (see *co-production* factors $\frac{1}{2}$) implies a re-normalisation factor equal to 4, because of the *global structure* of the System.

B) *Steady state conditions*, *with two equivalent* Source Terms [in D and E].

Eq. (3.21) may be now written as follows

$$1\cdot \dot{E}\, m(S) + 1\cdot \dot{E}\, m(F) + \dot{\Phi}_D + \dot{\Phi}_E = \dot{E}\, m(Z) + \left[6\cdot \dot{E}\, m(Y)\right] \quad (3.25) \quad \text{with} \quad \dot{\Phi}_E = \frac{1}{2}\dot{\Phi}_D \quad (2.26).$$

Eq. (3.25) is perfectly equivalent to Eq. (3.23). The only difference is the explicit presence of equivalent *static* Source Terms and a modified *re-normalisation factor* (equal to 6).

C) *Variable conditions* $\left[F(t), S(t)\right]$, *with two equivalent quasi-static* Source Terms [in D and E], and *two* Accumulation Terms [in D and E].

Eq. (3.21) now assumes the following structure

$$1\cdot \dot{E}\, m(S) + 1\cdot \dot{E}\, m(F) + \dot{\Phi}_D^* + \dot{\Phi}_E^* = \frac{\partial}{\partial t} A_D^* + \frac{\partial}{\partial t} A_E^* + \dot{E}\, m(z) + \left[6\cdot \dot{E}\, m(Y)\right] \qquad (3.27)$$

where $\dot{\Phi}_D^* = \dot{\Phi}_D$ (3.28) $\qquad \dot{\Phi}_E^* = \frac{1}{2}\dot{\Phi}_D$ (3.29)

and $A_D^* = A_D$ (3.30) $\qquad A_E^* = 2\cdot\left[A_E + \frac{1}{2}A_D\right]$ (3.31).

As it is easy to recognise from Eq. (3.31), the *Equivalent Accumulation Term* associated to the sub-System E does not only depend on its proper contribution $\left(A_E\right)$, however "amplified" by the particular sub-System productive structure $\left(\delta_{EE}=2\right)$, but also on a contribution (although reduced, $\varepsilon_{DD}=\frac{1}{2}$) due to the sub-System D, which *proceeds* it. The different "incidence" (or "weight") of the two respective *Equivalent Accumulation Terms* can be considered as an *evident* expression of the *different hierarchical value* of their corresponding "stocked" Emergy.

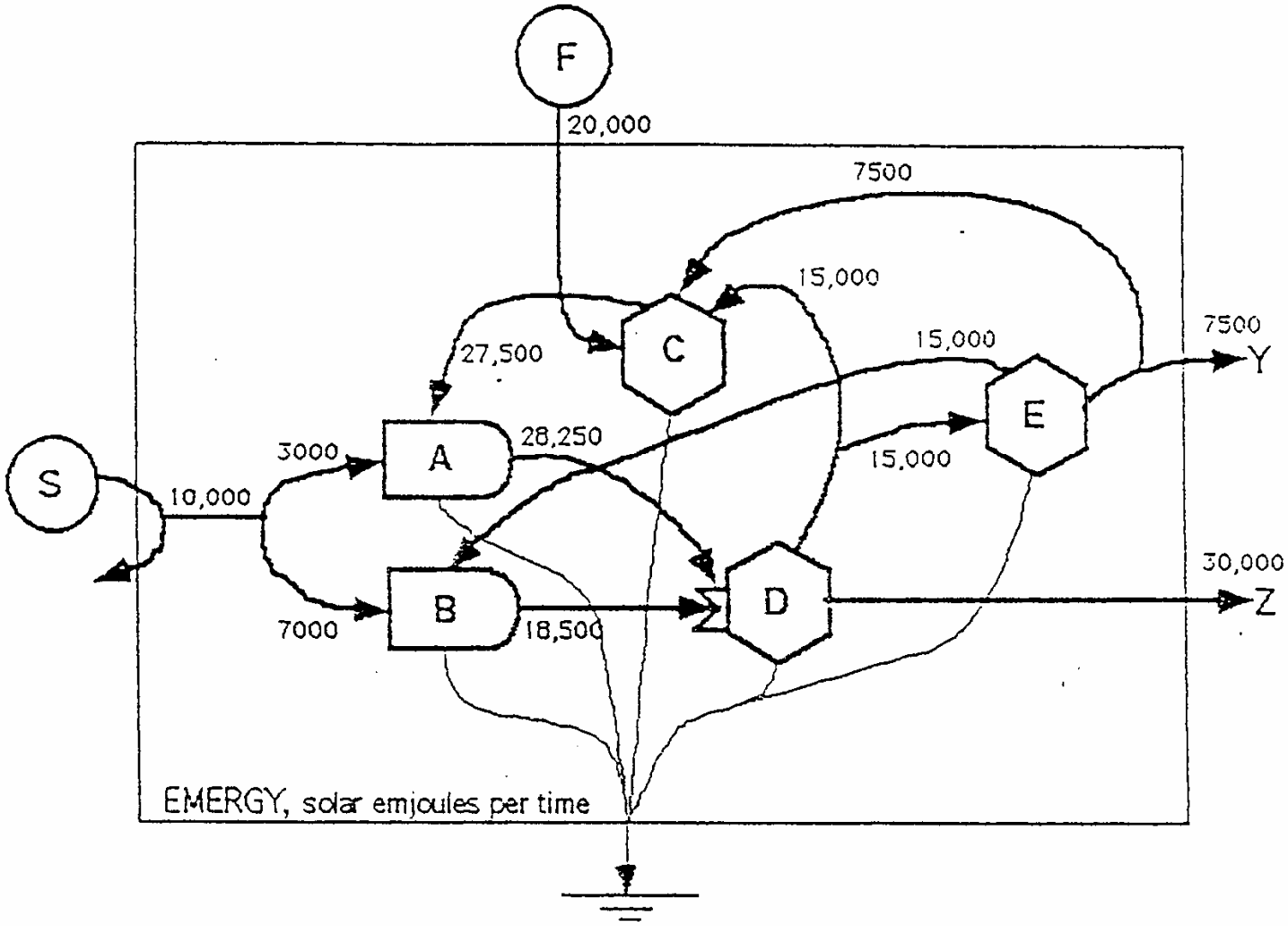


Fig. 4. A SUFFICIENTLY COMPLEX SYSTEM ANALYSED UNDER DIFFERENT CONDITIONS

These two general effects (sufficiently brought out even by the simple structure of Eqs (3.30) and (3.31) respectively) are still more relevant if we consider the following more articulated case.

D) *Variable conditions* $\left[F(t),S(t)\right]$, *with two* (*equivalent*) *quasi-static* Source Terms [in D and E], and *five* Accumulation Terms [in A,B,C,D and E].

In this case Eq. (3.21) will contain three additional *Equivalent Accumulation Terms* that may be expressed by an identical formal structure like the one already included in Eq. (3.22), that is

$$A^*_{D_i}\left(A_{D_1},A_{D_2},..,A_{D_n}\right)=\sum_{j=1}^{n}\varepsilon^*_{ij}\cdot A_{D_j} \qquad (3.32)$$

where the coefficients ε_{ij}^{*} express, through their specific "incidence", the contribution that each sub-System gives to the *i-th* considered *Equivalent Accumulation Term*, according to the corresponding *hierarchical position* of each one in the *connective structure* of the Whole System.
Therefore, if we consider Eq. (3.22) (in the light of Eq. (3.32)) we may easily recognise that the *Accumulated Emergy in the System* does not reduce just to the sum of all the *Tanks of Information*, but is *increased* by an additional contribution due to the *Information content* corresponding to *both* the *productive structure* and the *connective structure* of the System, in itself considered as *a Whole*.
Some similar considerations may be extended to the analysis of the *equivalent Source Terms*, as we will immediately see by considering the two following examples.
Let us start with a little more complicated case:
E) *Variable conditions* $\left[F(t), S(t)\right]$, *with two* (*equivalent*) *quasi-static + two dynamic* Source Terms [in D and E], and five Accumulation Terms [in A,B,C,D and E].
If we now suppose the presence of two *real dynamic* Source Terms (in addition to the quasi-static ones) in the sub-Systems D and E, their contribution will also extend their influence on the sub-Systems C, B and A. In fact, if we structure the considered Source Terms as follows

$$\dot{\Phi}_D(t) = \dot{\Phi}_{D,0}(t) + \chi_D \dot{\Phi}_{D,d}(t) \quad (3.33) \qquad \dot{\Phi}_E(t) = \dot{\Phi}_{E,0}(t) + \chi_E \dot{\Phi}_{E,d}(t) \quad (3.34)$$

the *dynamic* contributions $\dot{\Phi}_{D,d}(t)$ and $\dot{\Phi}_{E,d}(t)$ respectively, "amplified" by the *co-productive* structure of the pertinent sub-System (see coefficients χ_D and χ_E), determine two different effects:
- the appearance of three "equivalent" Source Terms also in sub-Systems C, B and A;
- the amplification of their specific contributions through feed-back pathways. In this case, in fact, we can not apply the 4th rule of Emergetic Algebra (e.g., in sub-Systems A and B), because these contributions are generated in different sub-Systems that *follow* the above mentioned ones.
In order to better analyse this last aspect, we prefer to consider an even more general case :
F) *Variable conditions* $\left[F(t), S(t)\right]$, *with two* (*equivalent*) *quasi-static + two dynamic* Source Terms [in D and E], plus *three dynamic* Source Terms [in A,B,C] and *five* Accumulation Terms [in A,B,C,D and E].
We will limit our consideration to only those aspects concerning the Source Terms and their effects.
As already shown in the case of Equivalent Accumulation Terms, each *Equivalent Source Term* (see Eq. (3.18)) may be structured in an analogous way

$$\Phi_k^{*}(u_1, u_2, .., u_m) = \sum_{r=1}^{n} \lambda_{kr}^{*} \cdot \Phi_r \qquad (3.35).$$

This expression is strictly *linear* if there are *no* amplification effects due to feed-back pathways. In this case the coefficients λ_{kr}^{*} express, through their specific "incidence", the *hierarchical contribution* that each sub-System furnishes to each considered Equivalent Source Term. In fact, analogously to what we show about Accumulation Terms, each *Source* that "follows" another one generally "amplifies" its proper contribution, not only because of the specific sub-System *productive structure*, but also because of its "higher" position in the *hierarchical structure* of the Whole System.
If, on the contrary, there are *amplification effects* due to feed-back pathways, Eq. (3.35) becomes (in general) non-linear because coefficients λ_{kr}^{*} do not only depend on the reciprocal position of the Sources, but also on their respective instantaneous values.
Under these conditions the System may become unstable (or not) according to some appropriate *corrective actions* which depend on the *self-organising* capacities of the System.
In the latter case, some *stable conditions* may actually be reached and these may correspond to different and persistent *oscillating behaviours* characterised by the fact of being in *stationary conditions*.

Before ending this paragraph, we want to point out another *fundamental aspect* that characterises the System in its complex, by emphasising that this aspect is also very well *reflected* in the General Balance Equation (3.21). This equation, in fact, does not exclusively give the *total* Emergy Power "coming out" from the System, but (in general) a higher *comprehensive contribution* due to the accounted *additional effect* constituted by the *Circulating Emergy Flow in the System.*
This quantity, if Eq. (3.21) is supposed to be already structured in its *standard form*[1], may be defined as

$$\dot{E}\, m_{circ} = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}\, m(y_l) - \sum_{l=1}^{p} \dot{E}\, m(y_l) \qquad (3.36)$$

and corresponds to the *Flow of Information* through the *feed-back pathways* of the System.
We may easily show this effect in steady state conditions (just for simplicity). In fact such a simple demonstration does not constitute a lack of generality. Let us then consider Eq. (3.25). Its second member is exactly equal to the sum of *two* contributions: *Output Emergy Flow* [which is equal to $\dot{E}\, m(Z) + \dot{E}\, m(Y)$] and *Circulating Emergy Flow* [which is equal to $5 \cdot \dot{E}\, m(Y)$].

4. Presuppositions for a Mathematical formulation of the "Maximum Em-Power Principle"
As initially mentioned, in this paper we will not (expressly) deal with the mathematical formulation of the Maximum Em-Power Principle, but only with its most correct presuppositions for such a possible formulation. To this purpose it is worth pointing out some *basic results* already carried out:
(i) a *rigorous* and *general definition of Emergy*, which is valid in conditions however variable;
(ii) a general *Global Balance Equation* that may be assumed to be as a fundamental presupposition «Toward..» such a researched formulation.
In addition, it is worth recalling that the considered Global Balance Equation, while it accounts for input and output quantities as a *global result* of the different *co-injective or co-productive* sub-System structures, it contemporarily accounts for *three* distinct additional contributions:
- *Accumulation Terms*, amplified by *both* the *productive* and *connective* structure of the System;
- *Source Terms*, characterised by similar (*productive* and *connective*) amplification effects;
- *Circulating Emergy Flow*, which quantifies the Flow of Information through feed-back pathways.
It is then clear that these three mentioned aspects (besides those pertaining to input/output quantities) are already sufficient to understand that the Maximum Em-Power Principle implies *optimum working conditions* (for the Whole System) which in general do not correspond to maximum efficiency conditions for each sub-System (each one in itself considered), but to those *particular* and *local* "*working conditions*" that *factually* realise the reciprocal "best fit" among sub-Systems in order to maximise the *global behaviour* of the System as a Whole.

5. Conclusions
Although this work is to be considered as still being in progress (in particular as far as the hypothesised final result is concerned), its logical bases and corresponding main achieved results seem to constitute valid presuppositions which may *orientate* future attempts at formulating a mathematical version of the "Maximum Em-Power Principle", at least under simplified conditions (steady state and stationary conditions).
At the same time, the work done up to now (and here presented in very synthetic form) should be already sufficient to understand that we are trying to mathematically express *something* which is (very likely) more than a *Thermodynamic* Principle.

[1] In this form each input and internal contribution to the System has its corresponding effect *directly* and *exclusively* expressed in terms of output quantities. Under these conditions we always have that $\left(\beta_l^* \cdot \beta_l\right) \geq 1$, for l = 1,2,… p.

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