Second Biennial Emergy Evaluation and Research Conference - Gainesville **(***USA, Florida***) -** *September 20-22, 2001*
see also web-site: www.emergysystems.org

Mathematical Formulation of the Maximum Em-Power Principle

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Abstract. **After having briefly recalled the main results shown in the previous Conference (a rigorous definition of Emergy in mathematical terms and a general mathematical formulation of the Emergy Balance equation) the paper presents an extremely general formulation of the Maximum Em-Power Principle. Such a formulation allows us to analyze in particular (but not exclusively):**
i) in what sense the "maximum" flow has to be understood, not only in steady state conditions but also in stationary and variable conditions;
ii) what reciprocal role Transformity and Exergy play in maximizing such a flow;
iii) in what sense such an extremely General Principle can be interpreted (but only reductively) as a Thermodynamic Principle.
The last aspect points out the different and wider presuppositions of the M. Em-P. Principle (as a "Thermodynamic" Principle) in comparison with the other Thermodynamic Principles (especially the First and the Second ones) and paves the way to a clear answer to the (apparently) contradictory assertions between the *Maximum* Em-Power Principle and the *Minimum* Action Principle.

1. Introduction

This paper expressly deals with the mathematical formulation of the Maximum Em-Power Principle. To this purpose it is worth starting from the basic results already achieved and presented at the First Emergy Research Conference [10], which constitute the most correct presuppositions obtained for such a possible formulation. Let us recall two of them very synthetically: a general mathematical definition of Emergy and the structure of the global Emergy Balance Equation.

i) A rigorous and *general definition of Emergy*, valid in whatever variable conditions, can be given by the following expression

$$Em^*(t) = \int_{-\infty}^{t} \dot{E}x_{eq}(\tau)d\tau \qquad (1.1)$$

where $Ex_{eq}(\tau)$ is defined as

$$Ex_{eq}(\tau) = \int_{D^*(\tau)} c(x,y,z,\tau)\cdot\rho(x,y,z,\tau)\cdot ex(x,y,z,\tau)d_3V \qquad (1.2)$$

and $\dot{E}x_{eq}(\tau)$ is the instantaneous *equivalent* Exergy Power used up during the process of generating a specific product.

Eqs. (1.1) and (1.2) translate, in mathematical terms, the general definition of Emergy given by H. T. Odum as «the total solar equivalent *available Energy* directly and indirectly used up to generate a specific form of Energy (or product)». In fact, the coefficient c which appears in Eq. (1.2) is a *dimensional* structural factor (whose dimensions are sej/J, that is solar emergy joules per Joule) and depends, among other things, on co-injection or co-production factors. It is thus defined in such a way as to *summarize all the rules of Emergetic Algebra* (this is the reason for the term "equivalent"); $D^*(\tau)$ is the Domain of integration which defines the quantity of the considered matter, $\rho(x,y,z,\tau)$ is the mass density, $ex(x,y,z,\tau)$ is the specific Exergy, while Newton's "dot" notation in Eq. (1.1) stands for the *total derivative* with respect to time.

Therefore, when we assume a Lagrangian perspective and steady state conditions, Eqs. (1.1) and (1.2) define the traditional and usual concept of Emergy, while under *whatever variable conditions* they define Emergy in its widest and most general conception.

ii) A general *Global Balance Equation* for any System made up of n sub-systems can be written as follows

$$\sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}\, m(u_j) + \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}\, m(y_l) \qquad (1.3)$$

where $\Phi_k^*(u_1, u_2, .., u_m) \left[\frac{sej}{\sec} \right]$ is the "*equivalent*" Source Term[1] relative to the *k-th* sub-System, which is expressed as a linear combination of all the n *real* sub-System contributions

$$\Phi_k^*(u_1, u_2, .., u_m) = \sum_{r=1}^{n} \lambda_{kr}^* \cdot \Phi_r \qquad (1.4),$$

while $A_{D_s(t)}$ is the *Global Accumulation Term* due to all the distinct contributions of the n sub-systems, thus it is given by an *appropriate sum* of n *equivalent* accumulation terms, each one (in turn) expressed as a linear combination of all the n *real* sub-System contributions

$$A_{D_s(t)} = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot A_{D_i}^*\left(A_{D_1}, A_{D_2}, .., A_{D_n}\right) = \sum_{i=1}^{n} \delta_i^* \cdot \delta_i \cdot \sum_{j=1}^{n} \varepsilon_{ij}^* \cdot A_{D_j} \qquad (1.5)$$

where

α_j, β_l, are the *co-injection* and *co-production* coefficients for each sub-System

α_j^*, β_l^* are their associated re-normalization factors (referred to the Whole System)

γ_k, δ_i are the pertinent "weights" in the corresponding sub-system balance equation

γ_k^*, δ_i^* are the associated re-normalization factors

$\lambda_{kr}^*, \varepsilon_{ij}^*$ are the specific incidence coefficients.

Moreover, it is worth recalling that the considered Global Balance Equation (1.3), in addition to accounting for input and output quantities as a global result of the different *co-injective or co-productive* sub-System structures, contemporarily accounts for three distinct additional contributions:

- *Accumulation Terms*, amplified by both the *productive* and *connective* structure of the System;
- *Source Terms*, characterized by similar (*productive* and *connective*) amplification effects;
- *Circulating Emergy Flow*, which quantifies the Flow of Information through feed-back pathways.

The latter quantity, if Eq. (1.3) is already structured in its *standard form*[2], may be defined as

$$\dot{E}\, m_{circ} = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}\, m(y_l) - \sum_{l=1}^{p} \dot{E}\, m(y_l) \qquad (1.6)$$

and corresponds to the *Flow of Information* through the *feed-back pathways* of the System.
On the basis of such presuppositions we will firstly try to answer a fundamental question.

2. **What exactly is Maximum?**

This is certainly not a trivial question. In fact the general *Global Balance Equation* (1.3), which is valid for any Complex System, may be written in several forms by organizing the different terms in such a way as they could appear, according to specific exigencies, either on the first or on the second side of the equation. Let us consider some of these forms.
In addition to Eq. (1.3) which, restructured as follows through Eq. (1.6), may be denominated as form A

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[1] Such a concept, already introduced in Ref. [10], will be analyzed in detail in the companion paper [18] presented in this Conference, especially as far as its profound meaning and wide consequences are concerned.

[2] In this form each input and internal contribution to the System has its corresponding effect *directly* and *exclusively* expressed in terms of output quantities. Under these conditions we always have $\left(\beta_l^* \cdot \beta_l\right) \geq 1$, for l = 1,2,… p.

A) $$\sum_{j=1}^{m}\alpha_j^*\cdot\alpha_j\cdot\dot{E}\,m(u_j)+\sum_{k=1}^{n}\gamma_k^*\cdot\gamma_k\cdot\Phi_k^*(u_1,u_2,..,u_m)=\frac{\partial}{\partial t}A_{D_s}(t)+\dot{E}\,m_{circ}+\sum_{l=1}^{p}\dot{E}\,m(y_l)\qquad(2.2),$$

we could also consider other main forms, for instance the following ones:

B) $$\sum_{k=1}^{n}\gamma_k^*\cdot\gamma_k\cdot\Phi_k^*(u_1,u_2,..,u_m)=\frac{\partial}{\partial t}A_{D_s}(t)+\dot{E}\,m_{circ}+\sum_{l=1}^{p}\dot{E}\,m(y_l)-\sum_{j=1}^{m}\alpha_j^*\cdot\alpha_j\cdot\dot{E}\,m(u_j)\qquad(2.3)$$

C) $$\sum_{k=1}^{n}\gamma_k^*\cdot\gamma_k\cdot\Phi_k^*(u_1,u_2,..,u_m)+\sum_{j=1}^{m}\alpha_j^*\cdot\alpha_j\cdot\dot{E}\,m(u_j)-\sum_{l=1}^{p}\dot{E}\,m(y_l)=\frac{\partial}{\partial t}A_{D_s}(t)+\dot{E}\,m_{circ}\qquad(2.4)$$

D) $$\sum_{k=1}^{n}\gamma_k^*\cdot\gamma_k\cdot\Phi_k^*(u_1,u_2,..,u_m)+\sum_{j=1}^{m}\alpha_j^*\cdot\alpha_j\cdot\dot{E}\,m(u_j)-\sum_{l=1}^{p}\dot{E}\,m(y_l)-\dot{E}\,m_{circ}=\frac{\partial}{\partial t}A_{D_s}(t)\qquad(2.5)$$

which do not evidently exhaust all the various other possibilities.

If we now consider the form A as the *reference structure* in order to formulate the M.Em-P.P., this assumption is equivalent to saying that "the System tends to maximize the sum of input and autogenerated Emergy flows (first side) in order to maximize the rate of accumulated Emergy together with circulating and output Emergy flows (second side)".

Analogously, if we take the other considered forms as reference structure, the pertaining formulation of the M.Em-P.P. would respectively assert that:

B) "The System tends to maximize the autogenerated Emergy flow (first side) in order to maximize the rate of accumulated Emergy, together with circulating and output Emergy flows, at net of the equivalent input one (second side)";

C) "The System tends to maximize the autogenerated Emergy plus the net input/output Emergy flows (first side) in order to maximize the rate of accumulated and circulating Emergy in the system (second side)";

D) "The System tends to maximize the autogenerated Emergy plus the net input/output Emergy flows at net of circulating Emergy flow (first side), in order to maximize the rate of accumulated Emergy in the system (second side)".

Such different possible statements allow us to reformulate the initial question in a more correct form: what is the mathematical structure that is potentially more suitable to best translate Odum's formulation of the Principle under consideration?

We will now try to show that form B is the most suitable starting structure in order to formulate such a *general reference principle* for self-organizing systems which is known as Maximum Em-Power Principle.

3. *Useful* Emergy and *processed* Emergy

In many publications H.T. Odum repeatedly asserts that "Em-power" has to be understood as "*useful* Emergy power". This could lead us to think (erroneously) that the Emergy Balance in form A is the one which is more suitable to represent such a concept, especially if we consider the terms which appear on the second side of Eq. (2.2). But it is also evident that, if the System has all the Source Terms $\Phi_k^*(u_1,u_2,..,u_m)$ equal to zero, it is intrinsically unable to organize input resources, so that it simply transforms the equivalent input Emergy flow into other forms of flow, without any incremental Emergy contribution that might be really *useful* to improve *either* its internal structure *or* its external Emergy products. It is not a self-organizing system at all, but it is only a mechanical *transducer*. In fact, under such conditions, all the pertinent co-production coefficients (β_l) are all equal to 1, as well as the corresponding re-normalization factors (β_l^*). Thus Eq. (1.6) implies that

$$\dot{E}\,m_{circ}=\sum_{l=1}^{p}\beta_l^*\cdot\beta_l\cdot\dot{E}\,m(y_l)-\sum_{l=1}^{p}\dot{E}\,m(y_l)=0\qquad(3.1).$$

Consequently: a) In *steady state* conditions such a system merely transfers input Emergy flow directly into output; b) In *variable conditions* input Emergy flow is simply transformed into the rate of Accumulated Emergy and output Emergy flow, but without showing any organizing

phenomenon (it acts like a mass anchored to a spring and contemporarily subjected to an external forcing energy source). In addition, if the output flow is persistently equal to zero, the system reduces to an Emergy storage (which evidently cannot be defined as a self-organizing system).
We may thus conclude that a *self-organizing* System (in its most meaningful sense) is only the one that "manages" input resources by increasing them by an additional positive and *net* contribution which is the only really *useful* one to *increase* both its rate of Accumulated and Circulating Emergy flow and its output Emergy flows.
In other words, what is really "useful" is not simply an input/output Emergy transfer, but the *net* contribution given by the internal generation process of Emergy which, through an appropriate organization of input resources, finalizes everything to a *net increase* of Emergy both in terms of rate of Accumulation, Circulation and output Production.
It should now be clear that the Balance Equation in form B is the one which is specifically indicated to express Odum's findings. At the same time, if we take into consideration that, apart from the Emergy Source Term, all the other forms of Emergy are involved in such a (*re*)*organization* process, we may consequently name them by a comprehensive term such as "processed" Emergy flows (obviously each flow is understood as accounted for with its own specific algebraic sign).
We can now propose the researched formulation of the M.Em-P. Principle.

4. Mathematical formulation of the Maximum Em-Power Principle

Eq. (2.3) can be at first re-organized in a more synthetic and compact form. In fact such characteristics, as already pointed out in Ref. [10], are particularly necessary in order to clearly answer the question as to whether the Maximum Em-Power Principle is really a Thermodynamic Principle or not. At the same time such a formulation should also be structured in such a way as:

i) to facilitate its comparison with the structure of the other Thermodynamic Principles (especially the First and the Second ones);
ii) to include the most general possible conditions pertaining to very Complex Systems, concerning both their time evolution behavior and space structure organization.

The latter consideration especially refers to the fact that, if we want to study a living organism (e.g. a plant or an animal) or even an organ (e.g., the liver or the brain) as a whole, the presence of billions and billions of cells suggests we consider a *continuous* system rather than a discrete one made up of n parts. This implies that Eq. (2.3) becomes more general if re-written in terms of integrals instead of summations. However, even if formulated in such a way, it can always be easily reduced into discrete terms, if necessary. And even if there might be some discontinuity conditions, they can always be dealt with in the frame of Lebesques' theory of integrals (Ref. [14]), which surely covers all the cases usually considered in Literature on the subject.
Under such assumptions, Eq. (2.3) can be re-written in the following synthetic and compact form

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \qquad (4.1)$$

where

φ_v^* = the "*equivalent*" Source Term per unit volume (see Eq.(1.4) in discrete terms);

Γ = the *local* structural amplification and re-normalization factor (corresponding to the product of the coefficients γ_k^* and γ_k in the case of discrete form (see Eq. (2.3)), which also accounts for the structural variations with time;

$$em_v^* = em_{v,m} + em_{v,q}^+ + em_{v,w}^+ \qquad (4.2)$$

in which

$em_{v,m} = C \cdot \rho \cdot ex$ is the Emergy per unit volume associated to the mass

(thus transportable by mass flows)[3]

$em^{+}_{v,q}$ = the Emergy per unit volume associated to heat source terms

$em^{+}_{v,w}$ = the Emergy per unit volume associated to work source terms[4].

Under such conditions the Maximum Em-Power Principle may be mathematically formulated as follows

$$\int_{D^*(t)} \Gamma \varphi^*_v d_3V = \frac{d}{dt} \int_{D^*(t)} em^*_v d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (4.3)$$

that is valid for any Domain (D^*) belonging to Universal Space ($S_U(t)$). Such a principle, in the light of the previous considerations, may be verbally formulated as follows: "Every System tends to organize its internal structure to generate progressively increasing spring-Emergy levels in order to maximize the flow of processed (or "useful") Emergy".

5. General considerations on the Mathematical Formulation of the M. Em-Power Principle

Formulation (4.3) may be seen as constituted by three parts:

1st part: $$\frac{d}{dt} \int_{D^*(t)} em^*_v d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (5.1)$$

which corresponds to the usual definition of the Principle in terms of phenomenological *effects*: "Every System tends to maximize the flow of processed (or "useful") Emergy";

2nd part: $$\int_{D^*(t)} \Gamma \varphi^*_v d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (5.2)$$

which points out the internal *causes* of such effects: "Every System tends to organize its internal structure for progressively increasing spring-Emergy levels";

3rd part: $$\int_{D^*(t)} \Gamma \varphi^*_v d_3V \overset{\rightarrow}{=} \frac{d}{dt} \int_{D^*(t)} em^*_v d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (5.3)$$

which emphasizes that its mathematical structure is a logical consequence of the first two mentioned parts and points out the existing direct relationship between internal *causes* (5.2) and phenomenological *effects* (5.1). The symbol $\overset{\rightarrow}{=}$ has been specifically introduced to emphasize the versus of the equivalence which goes *from causes to effects*.[5] Thus an alternative verbal formulation which is able to include *both* the first and the second side of Eq. (5.3) could be the following one: "Every System tends to maximize its internal structure level in order to maximize its Spring-

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[3] For the sake of generality, in open systems the specific mass Exergy is defined as $ex = (h - T_0 s) + \frac{1}{2}v^2 + gz$, that is it includes the kinetic and potential terms.

[4] The apex "+" indicates a positive quantity when *furnished* by the System, while its algebraic sign specifies the exact versus. Obviously we may use, if needed, all the consistent transformations such as $-A^+ = +A^-$ and so on. Such a convention is particularly useful for a successive comparison with the Second Principle formulated in terms of Exergy Balance Equation (see par. 6). Generally speaking we could say that the convention suggests the *perspective* according to which one should evaluate the actual versus of each considered flow.
Moreover, the term pertaining to work is understood as including a specific contribution due to pressure work such as $\int_{D^*(t)} \frac{\partial p}{\partial t} d_3V$ in order to have the same expression for specific mass Exergy (ex) both in the volume and surface integrals and in the case of both a Eulerian and Lagrangian description. At the same time the System is assumed to be subjected to conservative force fields which are supposed to be constant in time (as usually happens).

[5] Even if such a symbol is not adopted in writing the following equations, it will always be understood as being substituted by the *convention* according to which the equations will be (generally) written: the *left* side will represent the *causes* and the *right* side will represent the corresponding *effects*. Such a physical-mathematical convention (*from left to right*) is in some way analogous to the *topological* convention adopted in Emergy Analysis *System Diagrams.*

Emergy Flow which, in turn, maximizes the flow of processed Emergy". In addition it is also worth pointing out that Eq. (4.3) (or Eq. (5.3)) expresses a *tendency* Principle. This implies that:

i) the symbol *Max* has to be considered, in general, as an *Extremum*;

ii) thus in the long run, Eq. (4.3) can be also written as follows

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \geq 0 \quad , \quad \forall D^*(t) \subseteq S_U(t) \qquad (5.4)$$

where the sign equal holds only when the maximum is actually achieved;

iii) for very complex systems the dynamic behaviour is usually controlled by very high time constants, so that the slope of the increasing trend could be so slow that, in a given time-space window of analysis, it would be possible to assume that

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \cong 0 \qquad (5.5).$$

It is also worth mentioning that the previous expression (4.3) constitutes the formulation of the M. Em-P. P. in its *general version* that could also be said *in a weak sense*. In fact it is also possible to consider another and more cogent definition (*in a strong sense*) as follows

$$\frac{d}{dt} \int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d^2}{dt^2} \int_{D^*(t)} em_v^* d_3V \geq 0 \quad , \quad \forall D^*(t) \subseteq S^* \subset S_U(t) \qquad (5.6)$$

which is valid however only for particular subsets (S^*) of Universal Space S.

On the basis of the previous results we can now analyze the relationship between the M.Em-P.P. and the other Thermodynamic Principles.

6. The Second Principle in the light of the M. Em-P. Principle

As an introductory aspect let us consider a simple example: a Complex System made up of n discrete sub-systems. In this case Eq. (2.3) may be re-written as follows

$$(1+\varphi^*) \cdot \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) = \frac{\partial}{\partial t} A_{D_s}(t) + \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l) \qquad (6.1)$$

where, on the basis of Eqs. (1.3) and (1.4), we have defined Φ^* as follows

$$\Phi^* = \sum_{k=1}^{n} \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, .., u_m) = \varphi^* \cdot \sum_{j=1}^{m} \alpha_j^* \cdot \alpha_j \cdot \dot{E}m(u_j) \qquad (6.2)$$

that is the comprehensive "*equivalent*" Source Term (Φ^*) has been expressed in terms of the equivalent input Emergy flow, through an equivalent specific amplification (or generative) efficiency φ^*. Such a procedure is always possible on the basis of the n Emergy Balance equations describing the n considered sub-systems (see also Ref. [18]).

If we now assume, for the sake of simplicity, that the System is in steady state conditions and has just one input, we get $(1+\varphi^*) \cdot \dot{E}m(u) = \sum_{l=1}^{p} \beta_l^* \cdot \beta_l \cdot \dot{E}m(y_l)$ (6.3).

If (for ulterior simplicity) we assume all the co-production coefficients to be equal ($\beta_l = 1/p$) and renormalization factors equal to 1 ($\beta_l^* = 1$), we have

$$Tr(y_l) = Tr(u) \cdot \frac{Ex(u)}{Ex(y_l)} \cdot (1+\varphi^*) \qquad (6.4).$$

Eq. (6.4) has a fundamental importance because it shows that the Transformity of any output quantity is an amplification of the input Transformity as a consequence of *two* distinct reasons:

i) the dissipation of Exergy due to the Second Thermodynamic Principle ($Ex(y_l) < Ex(u)$);

ii) the amplification factor $(1+\varphi^*)$, which indicates the global gain specifically due to the generation of Emergy on behalf of internal Source Terms.

But what is really now worth pointing out is that, for very Complex Systems characterized by a lot of internal co-generation and/or interaction processes, the contribution given by the amplification factor $(1+\varphi^*)$ is generally *much higher* than the one due to Exergy losses. In addition, what is even more important is that the amplification due to *Emergy* Source Terms is *in-dependent* from the other one: the former can also be present both in ideal Systems (that is even if Exergy losses were absent) and in real Systems (even if characterized by an increase of Exergy).
This result can be considered as a first indication of the fact that the M.Em-P.P. is *independent* from the Second Principle and, what is even more important, it indicates the *effective reason* for the increasing order in self-organizing Systems, whereas the (habitually) associated loss of Exergy constitutes only a *concomitant circumstance*.
Such a conclusion can be easily (and more rigorously) drawn by starting from a very general point of view, that is by starting from the general formulation (4.3) of the M.Em-P. Principle (as we will see in par. 9). Let us now consider another important aspect: how it is possible to obtain the traditional Exergy Balance Equation from the one that expresses the M.Em-P. Principle. If, in fact, we consider Systems described only in terms of a traditional Thermodynamic approach (that is by neglecting any Emergy Source term, although ever present), our description reduces either to the classical Exergy Balance Equation or the Energy Balance Equation, according to the specific *distinct assumptions* considered in the two different mentioned cases.
The Exergy Balance Equation may be obtained by remembering that the irreversibility flow terms (mainly due to internal losses, but also to external ones) which can be expressed as

$$\frac{d}{dt}\int_{D^*(t)} em^*_{v,irr}\, d_3V = \frac{\partial}{\partial t}\int_{D^*(t)} C\cdot\rho\cdot ex^*_{irr}\, d_3V + \int_{\partial D^*(t)} C\cdot\rho\cdot ex^*_{irr}\, d_2S \qquad (6.5),$$

are generally neglected in Emergy Analysis on the basis of the assumption that their specific co-production coefficients are equal to zero (see Ref. [10]). If, vice versa, we continue to consider such contributions, but we distinguish between their *conceptual existence* and the fact that their pertinent local co-production coefficients are everywhere *numerically equal to zero* ($c_{irr}(x,y,z,\tau)=0$) (ib.), we may recognize that the Exergy Balance Equation can be thus easily obtained from the Maximum Em-Power Principle by assuming that all the co-injection and co-production factors are equal to 1. Such an assumption corresponds to considering *independent inputs* and all the *outputs as splits*. Under these conditions, and without any Emergy Source Term ($\Phi^*=0$), Eq. (4.3) becomes, in explicit terms

$$0 = \frac{\partial}{\partial t}\int_{D^*(t)}\rho\cdot(h_{kz}-Ts)\cdot d_3V + \int_{\partial D^*(t)}\rho\cdot(h_{kz}-Ts)\cdot v_{rn}\, d_2S + \int_{D^*(t)} \dot{q}^+_v\,\theta\cdot d_3V + \int_{\partial D^*(t)}\dot{q}^+_s\theta\, d_2S +$$
$$+ \int_{D^*(t)}\dot{w}^+_{v,p}\, d_3V + \int_{\partial D^*(t)}\dot{w}^+_s\, d_2S + \frac{\partial}{\partial t}\int_{D^*(t)}\rho\cdot ex^*_{irr}\, d_3V + \int_{\partial D^*(t)}\rho\cdot ex^*_{irr}\cdot v_{rn}\, d_2S \qquad (6.6)$$

which exactly expresses the Exergy Balance Equation, usually written as follows

$$\frac{\partial}{\partial t}\int_{D^*(t)}\rho\cdot(u_{kz}-Ts)\cdot d_3V + \int_{\partial D^*(t)}\rho\cdot(h_{kz}-Ts)\cdot v_{rn}\, d_2S =$$
$$= \int_{D^*(t)}\dot{q}^+_v\,\theta d_3V + \int_{\partial D^*(t)}\dot{q}^+_s\,\theta d_2S + \int_{D^*(t)}\dot{w}^+_v\, d_3V + \int_{\partial D^*(t)}\dot{w}^+_s\, d_2S - \frac{\partial}{\partial t}\int_{D^*(t)}\rho\cdot ex^*_{irr}\cdot d_3V - \int_{\partial D^*(t)}\rho\cdot ex^*_{irr}\cdot v_{rn}\, d_2S$$

(6.7.)[6] where θ is the generalized Carnot coefficient.

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[6] The difference between the presence of $h_{kz}=h+\frac{1}{2}v^2+gz$ in Eq. (6.6) and of $u_{kz}=u+\frac{1}{2}v^2+gz$ in Eq. (6.7) depends on the fact that the term $\dot{w}^+_{v,p}$ includes the pressure work mentioned in note 4, whereas the term $\dot{w}^+_v$ does not.