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Emergy Analysis as the First Ordinal Theory of Complex Systems

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ABSTRACT

The linguistic-mathematical tools developed to give an appropriate mathematical formulation to the Maximum Em-Power Principle, together with the already shown validity of the Rules of Emergy Algebra under variable conditions, suggested a possible development of a General Ordinal Theory of Complex Systems. This can ideally be thought of as an appropriate "transposition" of theorems and concepts of the well-known "Systems Theory" (conceived in terms of traditional Differential Calculus) into a new Mathematical Theory of Ordinal Systems, where the latter are described and analyzed in terms of "Incipient" Differential Calculus.

The activity aimed at formally defining some basic concepts (such as "Ordinality", "Information", "Organization", etc.) led us to recognize an unexpected "similarity" between the concept of "*Information*" adopted and Odum's Transformity.

Further consequential developments of such an initial analogy led us to the conclusion that Emergy Analysis can be considered as being the *First* Ordinal Theory in the field of Thermodynamics of self-organizing Systems. This result definitely becomes evident when the basic cardinal quantities, which in Emergy Analysis are understood in an "ordinal sense", assume their explicit and appropriate formal expressions corresponding to their Ordinal nature.

In particular, in such a context the Rules of Emergy Algebra can adherently be seen as specific Rules of "genesis and transfer of Ordinality", when the latter is accounted for in terms of its associated "*Ordinal Information*".

INTRODUCTION

The paper can be articulated in four parts: i) advantages of the Incipient Differential Calculus in Emergy Analysis; ii) advantages in other fields, in principle not strictly related to Emergy Analysis; iii) new challenges for the near future and usefulness of an Ordinal Systems Theory; iv) Emergy Analysis as the First Ordinal Theory of Complex Systems.

ADVANTAGES OF THE INCIPIENT DIFFERENTIAL CALCULUS IN EMERGY ANALYSIS

The Incipient Differential Calculus (IDC) was introduced in Emergy Analysis in the form of support. A support of a *linguistic nature*. In fact the "incipient" derivative was adopted to sustain the Rules of Emergy Algebra, first in steady state conditions, afterwards in variable conditions, in order to give the most general Mathematical Formulation to the Maximum Em-Power Principle (Giannantoni 2001b).

The "incipient" derivative, in fact, whose definition is here simply recalled (Giannantoni 2001c, 2002)

$$\frac{\tilde{d}^{\,q}}{\tilde{d}\,t^{q}}\, f(t) = \underset{\tilde{\Delta}\,t:0\to 0^{+}}{\tilde{\lim}} \cdot \left(\frac{\tilde{\delta}-1}{\tilde{\Delta}\,t} \right)^{\tilde{q}} \cdot f(t) \tag{1),}$$

because of its special properties, seems to be particularly indicated to express Emergy Concepts and, consequently, the associated *Incipient* Differential Calculus appears to be much more appropriate than the Traditional Differential Calculus (TDC), when dealing with Emergy Systems Dynamics.

Tab. 1 - Advantages of Incipient Differential Calculus in Emergy Analysis

Traditional Differential Calculus	*Incipient* Differential Calculus
1. LDE with variable coefficients: no explicit solutions, in finite terms and quadratures, for $n > 1$ (only expansion series)	1. LDE with variable coefficients: always have explicit solutions, in finite terms and quadratures, for any order n
2. Fractional LDE: no significant qualitative contribution: fractional derivatives can always be reduced to ordinary derivatives (Oldham & Spanier, 1974)	2. Fractional LDE: generation of explicit “binary” function (see co-production; no double counting) e. g. $\tilde{f}(t)^{1,1/(\tilde{2})}$
3. NLDE: no theorem of existence and uniqueness of solution; no explicit solutions (apart from some rare cases)	3. NLDE: generation of explicit “duet” functions (inter-action is only a particular case) e. g. $\tilde{f}(t)^{1,(\tilde{2})}$
4. No persistence of form (“drift” phenomena) $\frac{d}{dt} e^{\alpha(t)} = \alpha'(t) \cdot e^{\alpha(t)}$ $\frac{d^2}{dt^2} e^{\alpha(t)} = [\alpha'(t)]^2 \cdot e^{\alpha(t)} + \alpha''(t) \cdot e^{\alpha(t)}$	4. *Persistence of form* : generation of harmonic cords (see compositive Ordinality, e.g., in Feedbacks) $\frac{\tilde{d}^n}{\tilde{d}\, t^n} e^{\varphi(t)} = \left(\tilde{\varphi}'\right)^n \cdot e^{\varphi(t)}$ e. g. $\tilde{f}(t)^{1,1,[\tilde{n}]}$
5. “Cogent” (necessary and sufficient) conditions $f(t) = const \rightleftarrows \frac{d}{dt} f(t) = 0$	5. “Generative” (adherent and not sufficient) conditions $f(t) = const \underset{\not\longleftarrow}{\longmapsto} \frac{\tilde{d}}{\tilde{d}\, t} f(t) = 0$

The most important advantages of the IDC can be shown through a direct comparison between its basic properties and the corresponding properties of the TDC (see Tab. 1). Such a comparison, in addition, allows us to point out the main results achieved in Emergy Analysis by means of the IDC. In fact:
i) The introduction of fractional derivatives in the field of TDC does not add any significant qualitative contribution to the previous potentialities of linear differential equations (LDE). These derivatives, in fact, can always be reduced to ordinary integer derivatives (Oldham & Spanier, 1974). Vice versa, in the case of IDC, fractional derivatives generate a new class of functions, the “binary” functions (Giannantoni 2002, pp. 173-174) which allowed us to formally define the intimate properties of *co-products* and also show that, both in steady state and in variable conditions, there is no double counting in considering their contributions (Giannantoni 2001c, 2004a);
ii) While in the TDC there is no theorem of existence and uniqueness of solution concerning non-linear differential equations (NLDE) and, consequently, not even explicit solutions (apart from some rare cases (Davis 1960)), in the field of IDC, on the contrary, fractional derivatives generate a new further class of functions, the “duet” functions (Giannantoni 2001c, 2004a, 2004b). These enabled us to formally define the intrinsic properties of an Inter-action Process (both in steady state and variable

conditions), and in particular to show how this Process is capable of generating a correlative increase in Ordinality of output Emergy (Giannantoni 2004a);

iii) While in the field of TDC there is no *persistence of form*, that is there is no direct and generalized proportionality between a given function and its integer-order derivatives (aspect which will be referred to as "*drift phenomenon*")(Giannantoni 2004b), in the case of IDC there is always a *persistence of form* between any given function and all its derivatives, both of integer and fractional order (Giannantoni 2002, p. 176; 2004b). Both "drift phenomenon" (in the case of TDC) and "persistence of form" (in the case of IDC) are shown in Tab. 1 with reference to the exponential function (Giannantoni 2004b).

Such a persistence of form was exactly that property which enabled us to show why Feed-backs not only introduce a stability action, but also generates a more harmonic behavior in any given Process (Giannantoni 2004a). The afore-mentioned proportionality, in fact, is the basis for the generation of harmonic "chords" (like in music), which further increase the Ordinality of the Process (ib.).

All these properties, which in principle are also valid in any field of Physics (not only in Emergy Analysis), led us to think about the possibility of facing some "problematic" aspects (or even unsolved problems) in Classical Mechanics.

ADVANTAGES OF IDC IN OTHER FIELDS IN PRINCIPLE NOT STRICTLY RELATED TO EMERGY ANALYSIS

For the sake of brevity we will mention only three problems of Classical Mechanics: i) Mercury's Precessions; ii) Constant position of orbital planes; iii) the Three-body Problem. Let us examine them in rapid succession:

i) **Mercury's Precessions** have never received an acceptable explanation over 300 years of Classical Mechanics (whose specific mathematical language has always been based on the TDC) (Landau & Lifchitz, 1969). Vice versa, such *precessions* can be easily explained as being a consequence of the "drift phenomenon" associated to the (traditional) second order derivatives. In fact it is sufficient to reformulate the fundamental Laws of Classical Mechanics in terms of Incipient Derivatives (see Tab. 2), in order to get results which are in almost perfect agreement with the most recent astronomical data (Giannantoni 2001c, 2004b).

ii) **The constant position of orbital planes,** in Classical Mechanics, is a necessary consequence of a linguistic-mathematical nature: the adoption of TDC as a basic language, which necessarily leads to conservation Principles ((Landau & Lifchitz, 1966); see also the necessary and sufficient conditions in Tab. 2). The IDC, on the contrary, is a little more "flexible". In fact, if the first order incipient derivative of any quantity equals zero, this is no longer a sufficient condition to state the "conservation" of the considered quantity (Giannantoni 2002, pp. 64-65). Consequently, orbital "planes" of the planets can have a gyroscopic motion (Giannantoni 2004b). This aspect becomes even more evident if the planet is not considered as being a simple material point in space, but as forming a "binary" System together with the Sun (ib.).

Both these results encouraged us to face one of the most celebrated problems in Classical Mechanics: the "Three-body Problem".

iii) **The Three-body Problem** was proved to be *intrinsically unsolvable* in Classical Mechanics (Poincaré 1899). In fact it is described by an 18th-order system of ordinary differential equations, but admits only 2 first order *closed form* integrals (energy and areas)(Poincaré 1899, vol. 1, p. 253). Vice versa, in terms of incipient derivatives, the problem becomes perfectly solvable. The solution, always in *a closed form* (as explicitly desired by Poincaré (ib.)), is easily obtained when the three bodies are considered as forming a "ternary" System (such results will be published before the end of the year).

It is worth adding that the Three-body Problem is not only extremely important in Celestial Mechanics, but also in Quantum Mechanics (in particular in the case of molecules with more than two atoms). This aspect will be recalled at the end of the paper.

Tab. 2 - Advantages of Incipient Differential Calculus in other Fields

Traditional Differential Calculus	*Incipient* Differential Calculus
1. Mercury's Precessions: no acceptable explanation over 300 years of Classical Mechanics	1. Explanation as a "drift" phenomenon . It is sufficient to reformulate the fundamental Laws of Classical Mechanics in terms of Incipient Derivatives
$\vec{F} = \frac{d\,\vec{p}}{dt} \qquad \vec{M} = \frac{d\,\vec{b}}{dt}$	$\vec{F} = \frac{\tilde{d}\,\vec{p}}{\tilde{d}\,t} \qquad \vec{M} = \frac{\tilde{d}\,\vec{b}}{\tilde{d}\,t}$
2. Constant position of orbital planes (conservation Principles)	2. Gyroscopic motion of orbital "planes": "binary" Systems (Giannantoni, 2004b)
$\vec{b} = const \Longleftrightarrow \frac{d\,\vec{b}}{dt} = 0$	$\vec{b} = const \;\; \overset{\Longrightarrow}{\not\Longleftarrow} \;\; \frac{\tilde{d}\,\vec{b}}{\tilde{d}\,t} = 0$
3. The "Three-body Problem": *intrinsically unsolvable* in Classical Mechanics (Poincaré,1899): an 18th-order system of ordinary diff. eqs., with only 2 first order *closed form* integrals (energy and areas)	3. Perfectly solvable, in *a closed form*, when considered as a "ternary" System. Analogous consequences in Quantum Mechanics (in the case of molecules with more than two atoms)

The above-mentioned advantages of the IDC, both in Emergy Analysis, in Classical Mechanics and even at a more general level, suggested the idea of its possible application to those problems which are generally recognized as the most urgent in the Scientific Field and, at the same time, represent real challenges for the near future.

SCIENTIFIC CHALLENGES FOR THE NEAR FUTURE. USEFULNESS OF AN ORDINAL SYSTEMS THEORY

These challenges fundamentally concern: i) Weather-Forecasts; ii) Climate Change; iii) The necessity for a General Theory of non-linear Complex Systems.

i) **Weather-Forecasts** are extremely important, not only as a consequence of the very rapid increase in intensity and frequency of hurricanes, but also for all the atmospheric phenomena that could damage any sort of harvest. Its potential solution in the field of TDC shows intrinsic limitations as a consequence of the so-called Ljapounov's Time (Strogatz 2003, p. 244-245). This "time" is intrinsically defined by the mathematical model adopted (about 48 hours) and indicates the time interval after which any solution obtained progressively loses its proper physical sense (ib.).

Vice versa, in the case of IDC, there are no limitations related to Ljapounov's Time, because the corresponding mathematical model shows the total absence of "drift phenomena".

ii) **Climate Change,** understood as a global effect, probably really exists. However we are not able to reach a definitive conclusion on the subject, especially because we are only able to depict possible (and variegated) "scenarios", without any possibility of foreseeing any reliable dynamic behavior in the long term (50-100 years). In fact any mathematical model (always based on TDC) suffers from

limitations accounted for in Hadamard's Theorem: all the solutions are always non-linearly dependent on the initial conditions. Consequently, uncertainties about the initial conditions, together with round-off errors, end up by "destroying" the solution very rapidly in time.
Vice versa, models based on IDC no longer have those limitations related to Hadamard's Theorem, because the corresponding solutions are always linearly dependent on the initial conditions (2001c, 2004b, 2004c). This means that any desired precision can always be achieved (in the context of the solution obtained).
iii) **The necessity for a General Theory of non-linear Complex Systems**. In this respect it is worth noting that: a) on the one hand, there is a general tendency to increase computing power (e.g. from the present 10 Teraflops to 1 Petaflop within 2010). Such a tendency, however, is already facing "saturation" (e.g., power supply is reaching about 500 MW) (Rosato et al., 2004). At the same time the best performances expected (1 Petaflop) is still insufficient for extremely important research concerning *protein folding* (e.g., a protein made up of 2000 atoms, that is a very "elementary" protein, would require 10.000 years of continuous computing time (ib.)); b) on the other hand the problem is generally and constantly thought of as being solvable in terms of a *multiplication of quantities*, whereas it is only a problem of *Irreducible Quality* (Giannantoni 2002, chapter 12). This is the fundamental aspect which, although always subjacent to systems of NLDEs, is much more explicitly "ostended" by incipient derivatives.

All the afore-mentioned problems surely represent convergent reasons for researching a general approach to an Ordinal Systems Theory in terms of IDC. However, we may ask: how is it possible to "build" such an Ordinal Systems Theory? The answer to this question, articulated in four steps, will directly lead to the thesis of this paper.

EMERGY ANALYSIS AS THE FIRST ORDINAL THEORY OF COMPLEX SYSTEMS

The first basic problem is that of researching a general procedure in order to pass from the knowledge of any single process to the knowledge of the Whole Process.

Passage from a Single Process to the Whole Process

Every elementary process is in fact described by a non-linear differential equation, of order n_j, with variable coefficients, in the basic fractional derivative $1/m_j$, with a maximum degree of non-linearity expressed by l_j (Giannantoni 2004 c; see also Fig. 1b).

The explicit output "function" is characterized by; i) a *cardinality* (expressed by the symbol $f(t)$ raised to the traditional cardinal power q_j); ii) an *Ordinality* $(\tilde{l}_j)/(\tilde{m}_j)$, given by the basic fractional derivative $1/(\tilde{m}_j)$ multiplied by the maximum degree of non-linearity $(\tilde{l}_j)$; iii) and a *compositive Ordinality* $[\tilde{n}_j]$, which reminds us of the "musical" chord between the n_j components of the differential equation (Giannantoni 2004c).
The problem thus concerns the research of the unknown *Ordinal relationship* between cardinalities, Ordinalities, and compositive Ordinalities of all the elementary processes and the same concepts (*cardinality*, *Ordinality*, *compositive Ordinality*) pertaining to the output Function $F(t)$ of the Whole Process (see Fig. 1a)

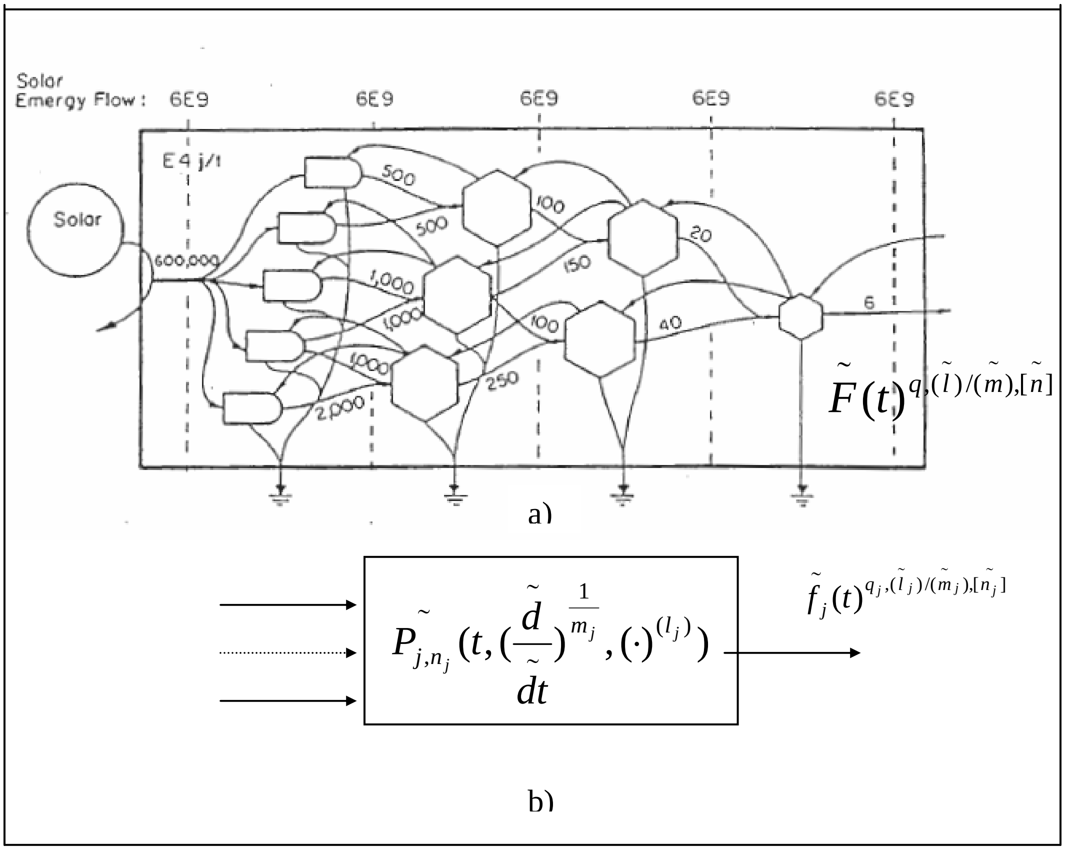

Fig. 1 – a) General scheme of a Complex System (Odum, 1994a);
b) mathematical model of any elementary process

It is rather evident that the problem is not very easy to solve. In fact there is no linear *Ordinal relationship* (in principle) between the elementary processes and the Process as a Whole. This means that we should need a sort of "guide" or, at least, a possible starting point. The latter could be then represented by the Traditional Systems Theory, although still based on the TDC. In other words, we could research a general procedure to transform the "Systems Theory", based on TDC, to an "Ordinal Systems Theory", based on IDC.

From the "Systems Theory" to an "Ordinal Systems Theory"

As is well known, the "Systems Theory" is the result of several contributions: i) Ljapounov's studies (1900-1918); ii) the Servo-mechanisms Theory, whose development (1940-50) was particularly influenced by the Second World War**;** iii) the Automatic Systems Theory (from the 1950's on) progressively favored by the development of digital technologies.

All these Theories are based on the Traditional Differential Calculus. So we thought about transposing all the theorems and concepts of such fundamental Theories in terms of the Incipient Differential Calculus, by obviously taking into account the *different properties* deriving from the adoption of *a posteriori* derivatives (d/dt) with respect to *a priori* derivatives ($\tilde{d}/\tilde{d}t$) (Giannantoni 2002, pp. 174-176).

During such a process of transposition (which has already reached a sufficiently advanced stage), a question arose spontaneously: what could ever the words "organization" and "information" possibly mean in such a new context, precisely because of their being thought of in *Ordinal terms*?
Exactly here, in such an attempt at answering this question, we discovered profound emerging relationships between those *completely new* concepts and Odum's Transformity. Let us examine them in rapid succession.

Ordinality, Organization, Information

Let's start from the concept of Ordinality. This concept expresses the fact that the Whole System is the result of *m* cooperative actions, amplified by *l* forms of non-linearity, apart from the harmonic consonance of all the *n* differential components.
If we now, by starting from this concept, focus on the *sole* Ordinal Relationships, by neglecting their pertinent instantaneous values, we get the associated concept of *Ordinal Organization*. This in fact can be defined as "the topological structure of the *Ordinal* Relationships, with their specific genetic priority, apart from their instantaneous cardinal values".
For example: if we consider the "circle" product ($\circ$)[1] between a *binary* function and a *duet* function, we get a "binary" function which interacts with itself in the form of a duet:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \circ [b_1, b_2] = \left[\begin{pmatrix} a_1 b_1 \\ a_2 b_1 \end{pmatrix}, \begin{pmatrix} a_1 b_2 \\ a_2 b_2 \end{pmatrix} \right] \qquad (2).$$

If we now leave the instantaneous cardinal values to one side and replace them by means of any alternative symbols (see Eq. (3)), we get the *Ordinal Organization* of the above-mentioned Process. In this way, in fact, we are left with the *sole* Ordinal relationships, with their associated genetic priorities, totally deprived of any *functional* character:

$$\begin{pmatrix} *_1 \\ *_2 \end{pmatrix} \circ [\lozenge_1, \lozenge_2] = \left[\begin{pmatrix} *_1 \, \lozenge_1 \\ *_2 \, \lozenge_1 \end{pmatrix}, \begin{pmatrix} *_1 \, \lozenge_2 \\ *_2 \, \lozenge_2 \end{pmatrix} \right] \qquad (3).$$

It is worth pointing out that Eq. (2) was explicitly indicated not only to facilitate the exposition of the concept of Organization (understood in Ordinal terms), but also to underline that its right hand side is not a traditional matrix. It is in fact an *Ordinal Matrix*, that is: a Matrix of Ordinal Relationships, where cardinalities *only* represent a simple *topological* support, as better indicated in Eq. (3).
The last concept of interest, *Ordinal Information*, could be thought of as an appropriate Indicator of the Complexity of the considered System. To this purpose we could assume that "binary" functions and "duet" functions, if made up of the same number of elements, are substantially "equivalent" in terms of Complexity. We may thus "transpose" all the "binary" functions into "duet" functions and so write

$$\textit{Ordinal Information} \overset{\Delta}{=} (\tilde{m})^{(\tilde{l})} \qquad (4).$$

In other words, *Ordinal Information* is an Indicator of the *Complexity* of the System, expressed in terms of *equivalent* "multiple duet" functions amplified by non-linear processes.
And now that the concepts of Ordinal Organization and Ordinal Information have been formally defined, we can easily show that the concept of Odum's Transformity, with its intrinsic *ordinal sense*, corresponds exactly to the concept of Ordinal Information. We could also recognize that Emergy Algebra is already Ordinal Algebra.

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[1] The symbol "$\circ$" represents a generalized form of product (termed as "circle product" (Giannantoni, 2002, p. 178)), whose result accounts for both cardinality and Ordinality of the given "factors".

Emergy Algebra as the First Ordinal Algebra

Let's start from the analysis of the three fundamental generative processes represented by Co-production, Inter-action and Feed-back (Giannantoni, 2004a)

a) Co-production. According to Odum's Rule, Emergy output is twice as much as Emergy input:

$$Em(y) = 2 \cdot Em(u) \tag{5.}$$

If the same rule (in steady state conditions) is expressed in terms of incipient derivatives (ib.), output Emergy results as being a "binary" function, which may be represented in different formal ways. In particular, it can always be expressed in terms of *cardinality* and *Ordinality* (see last term of Eq. (6))

$$\tilde{E}\,m(y) = \begin{pmatrix} \tilde{E}\,m(y_1) \\ \tilde{E}\,m(y_2) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \tilde{E}\,m(u) = \tilde{E}\,m(u)^{1,1/(\tilde{2})} \tag{6.}$$

However, the specific finalities we are concerned with suggest we adopt, for the sake of clarity, a different notation. On the other hand the *physical* meaning of Ordinality does not depend on the specific notation adopted. We can thus write

$$\tilde{E}\,m(y) = \tilde{E}\,m(u)^{1,1/(\tilde{2})} = [1/(\tilde{2})] ** \tilde{E}\,m(u) \tag{7}$$

where the symbol of "power" $(**)$ is evidently borrowed from FORTRAN Language, whereas $[1/(\tilde{2})]$ is correspondently ante-posed because Ordinality has a *logical priority* with respect to cardinality. The "double asterisk" $(**)$ can also be seen as an indication of a double "association product" $(*)$, the latter represented by a single asterisk. In this way, in fact, the symbology adopted also reminds us that there exists a *reciprocal conjugated* relationship between Ordinality and cardinality.

Moreover, we can also express Emergy in terms of Ordinal Information (or, better, the Emergy content of Ordinal Information):

$$\bar{E}\,m(y) = (\tilde{2})^1 * \tilde{E}\,m(u) \tag{8.}$$

In this case we have a simple "association product" (*) because Ordinal Information is always "associated" to the pertinent cardinality in one sole unidirectional way (it is like saying that the passage from Ordinality to Ordinal Information "absorbs" an "association product" (*)).

If we now compare Eq. (8) with Eq. (5), and we remember Odum's use of Transformity in an *Ordinal sense* (Giannantoni, 2004a), we can easily recognize that co-production output Emergy expresses the increased content of Ordinal Information that, as a consequence of the process, is associated to input Emergy.

b) Inter-action. According to Odum's Rule, output Emergy is proportional to the product of input Emergies

$$Em(y) = k_{\text{int}} \cdot Em(u_1) \cdot Em(u_2) \tag{9.}$$

If the same rule (in steady state conditions) is expressed in terms of incipient derivatives, output Emergy results as being a "duet" function (ib.). In addition, for the sake of generality, input Emergies can be thought of as being characterized by more articulated Ordinalities (see second term of Eq. (10)). If we then follow the same procedure already shown in the previous section, we can successively write

$$\tilde{E}\,m(y) = [\tilde{E}\,m(u_1)^{1,1/(\tilde{m}_1)}, \tilde{E}\,m(u_2)^{1,/(\tilde{m}_2)}] =$$
$$= [\tilde{E}\,m(u_1)\cdot\tilde{E}\,m(u_2)]^{1,2/(\tilde{m}_1\tilde{m}_2)} = \frac{2}{(\tilde{m}_1\,\tilde{m}_2)} **[\tilde{E}\,m(u_1)\cdot\tilde{E}\,m(u_2)] \qquad (10).$$

Furthermore, in analogy with the previous case, we can also express Emergy in terms of Ordinal Information (or better, as already said, the Emergy content of Ordinal Information), to get

$$\bar{E}\,m(y) = [\tilde{E}\,m(u_1)^{1,1/(\tilde{m}_1)}, \tilde{E}\,m(u_2)^{1,/(\tilde{m}_2)}] = (\tilde{m}_1\,\tilde{m}_2)^2 *[\tilde{E}\,m(u_1)\cdot\tilde{E}\,m(u_2)] \quad (11).$$

By comparing Eq. (11) with Eq. (9), and by still remembering Odum's use of Transformity in an *Ordinal sense* (Giannantoni, 2004a), we can easily recognize that Inter-action output Emergy represents the increased content of Ordinal Information that, as a consequence of this process, is associated to the product of input Emergies. In fact, the proportionality coefficient k_{int} in Eq. (9) has not always the same *unique* definite value, but depends on the characteristics of the particular Inter-action considered. In addition, the last term of Eq. (11) clearly shows that, even if k_{int} is indicated (in Eq. (9)) by means of "a scalar", in reality it has to be understood in an *Ordinal sense*. In fact it expresses the *increase of Ordinal Information* due to the Inter-action Process.

c) Feed-back. The same considerations can be made with reference to the Feedback Process. In fact, if the latter is characterized by a constant input $Em(u_0)$, its output Emergy at permanent regime is given by (ib.)

$$\tilde{E}\,m[y(t)] = W_0^{1,(\tilde{r}),[\tilde{n}+\tilde{r}]}\cdot\tilde{E}\,m(u_0) \qquad (12)$$

where

$$|W_0| \cong 1 \qquad (13).$$

As usual, Eq. (12) can equivalently be re-written as

$$\tilde{E}\,m[y(t)] = \{(\tilde{r}),[\tilde{n}+\tilde{r}]\} **W_0 \cdot\tilde{E}\,m(u_0) \qquad (14)$$

which, apart from the double "association product" $(**)$, coincides exactly with the expression of output Emergy in terms of Ordinal Information (characterized by a single "association product")

$$\bar{E}\,m[y(t)] = \{(\tilde{r}),[\tilde{n}+\tilde{r}]\} *W_0 \cdot\tilde{E}\,m(u_0) \qquad (15).$$

Eq. (15), when thought of in terms of Transformity, shows that the latter accounts for the increase in Ordinal content of Information due to two distinct contributions, both associated to the presence of the feedback chain: a multiple "duet" Ordinality $(\tilde{r})$, as a consequence of an incipient differentiation of order r and a *compositive* Ordinality $[\tilde{n}+\tilde{r}]$ due to an incipient integration of order $n + r$ (Giannantoni, 2004a).

The analysis of the three afore-mentioned Generative Processes (Co-production, Inter-action, Feed-back) suggests that the Rules of Emergy Algebra could then be seen as *Rules of Genesis and Transfer of Ordinality*.

Rules of Genesis and Transfer of Ordinality

The Rules of Emergy Algebra, in fact, can be subdivided in *two groups* and re-proposed in a different *sequence*, by always keeping the same formulation given by Prof. Brown (Brown, 1993; Brown & Herendeen, 1996)).

1st group: made up of Co-production, Inter-action and Feed-back. These can be seen as *Rules of Genesis of Ordinality* (in steady state conditions):

i) Co-production**:** "*By-products from a Process have the total Emergy assigned to each pathway*"

ii) Inter-action: "*Output Emergy of an interaction Process is proportional to the product of the Emergy inputs*" (Odum, 1994a)

iii) Feed-back**:** "*Emergy in feedbacks should not be double counted* ".

2nd group: made up of the First Rule and Split. These can be seen as *Rules of Transfer of Ordinality* (in steady state conditions) from one part of the System to another. In particular: from input to output (the former rule); from the main flow to the subdivided flows (the latter rule):

iv) First Rule: "*All Source Emergy to a Process is assigned to the Process's output* "

v) Split: "*When a pathway splits, the Emergy is assigned to each "leg" of the split based on their percent of the total Exergy flow on the pathway"* (ib.; Giannantoni, 2002).

At this stage, by remembering all the advantages previously shown due to the Incipient Differential Calculus, and in particular those concerning the concepts of Emergy and Transformity, we may ask: why do we not adopt an *explicit Ordinal notation* for the Generative Transformity (Tr_ϕ)?

RATIONAL AND CONCLUSIONS

The previous question is based on the following logical steps:

i) We already know that the concept of Transformity can be thought of as the product of two scalar factors (Giannantoni 2002, 2004a)

$$Tr = Tr_\phi \cdot Tr_{ex} \qquad (16)$$

where Tr_{ex} is the dissipative Transformity and Tr_ϕ is the generative Transformity.

The first one accounts for the losses of Exergy used up during the generation process of a given product or service. The generative Transformity, on the other hand, accounts for all the contributions to an ever-increasing content of *Ordinal Information* due to the various Generative Processes (Co-production, Inter-action, Feed-back);

ii) These two factors are, in principle, immixible, because they represent two *distinct physical* aspects. Thus they should be represented as being simply associated, for example as follows

$$Tr = (Tr_\phi, Tr_{ex}) \qquad (17);$$

iii) By remembering Odum's use of Transformity in an *Ordinal sense* (Giannantoni, 2004a), we could adopt an appropriate *ordinal notation* for Tr_ϕ. For example, by including it in round brackets:

$$Tr = [(Tr_\phi), Tr_{ex}] \qquad (18);$$

iv) The last step would only consist in adopting for Generative Transformity that *explicit notation* which precisely *emerges* from the IDC (based on the incipient or "generative" derivative ($\tilde{d}/\tilde{d}\,t$)):

$$Tr = [(\tilde{T}\,r_\phi), Tr_{ex}] \qquad (19).$$

As a conclusion, we can assert that: *if* Tr_ϕ is adherently replaced by $(\tilde{T}\,r_\phi)$, *Emergy Analysis becomes the First Ordinal Theory of Complex Systems*.

RELATED CONSEQUENCES

The explicit assumption of the Generative Transformity as an *Ordinal concept* evidently has some related consequences. We will only mention two of these which appear to be particularly interesting.

1. A renewed concept of Sustainability

Let us consider the scheme in Fig. 1 and suppose that the last hexagon on the right side represents an Energy Power Plant to be optimized. The standard procedure generally consists of two conceptual phases: i) the Power Plant is preliminarily optimized in terms of *efficiency*, on the basis of the First and Second Principles of Thermodynamics; ii) it is then optimized in terms of *use of resources* deriving from both the Environment, Economy, Human Labor and so on. In the case of a particularly complicated Whole System an iterative procedure can be usefully adopted.

In such an optimization procedure**,** the concept of *Sustainable Development* is understood as "*durable*" (or "*lasting*") Development. In other words, resources are managed in such a way as to enable the Plant, which is already optimized in terms of efficiency, to continue to work for long time to come.

However, if we organize both Plant and Resources so as to maximize the *hierarchical Ordinality level* of the Whole System, we will reach, at the same time, both *the optimum working point* for the Plant and, as a conjugated effect, the *minimum associated use of resources*.

Evidently both perspectives are fundamentally convergent toward the minimum use of resources. Nonetheless the latter optimizing procedure seems to more faithfully interpret Odum's concept of "Macroscope" which, in an *explicit* Ordinal perspective, could also be termed as "Holoscope", precisely because the Process is now seen as one sole irreducible entity, understood in intensive terms.

In such a case *Sustainability* would indicate the minimum use of resources, simply obtained as a *conjugated aspect* of the Maximum Ordinality level achieved.

Similar concepts can be illustrated with reference to another aspect, which is even more important than the previous one.

2. The possibility of conceiving New Ordinal Energy Processes

Let us consider, for example, the physical process (we) synthetically termed as "Photon-phonon Interaction". It could be described as follows (see also Fig. 2): at the first stages of chlorophyll synthesis, a photon, characterized by a wave length of 673 nm (that is a red photon), hits a molecule of H2O and separates Hydrogen from Oxygen.

In various world laboratories several experiments are being made to reproduce such a process, up to now without success. This could depend on our traditional conceptions (derived from Physics) about both photon, water and their interaction.

The *traditional interpretation*, in fact, considers: i) a photon as "pure" Energy (linear momentum without mass); ii) H2O molecule as a *functional* structure; iii) only *statistical* positions of the single atoms (because of the unsolvable Three-body Problem); iv) only "mechanical" *efficient* interaction.

An *interpretation in Ordinal terms*, vice versa, would consider: i) a photon as a "binary" System, made up of a positron and an electron following two co-axial spiraloidal trajectories (Giannantoni, 2001a); ii) H2O molecule as a "ternary" System; iii) the *correct* orbital trajectories deriving from the solution to the Three-body Problem; iv) all the involved *resonance* phenomena and their associated global *compositive* Ordinality.

On such bases the Process analysis will simply consist in a *release of* (*cardinal*) *Energy as a conjugated aspect of a new Ordinality level achieved*.

Such an alternative interpretation, substantially based on Transformity understood as an Ordinal concept, could evidently simplify the physical reproduction of such a Process, which, in turn, could give an enormous contribution (in cardinal terms) to the Energy Supply of the entire Earth and, at the same time, would be perfectly compatible with the Environment (in Ordinal terms).

In extreme synthesis, all such results could be seen as a simple consequence of the introduction of a different formal Language, specifically adopted to support Emergy Analysis, so that the latter could more and more widely diffuse a different way of *thinking* and, consequently, of *acting* (in an ever-increasing *Ordinal sense*).

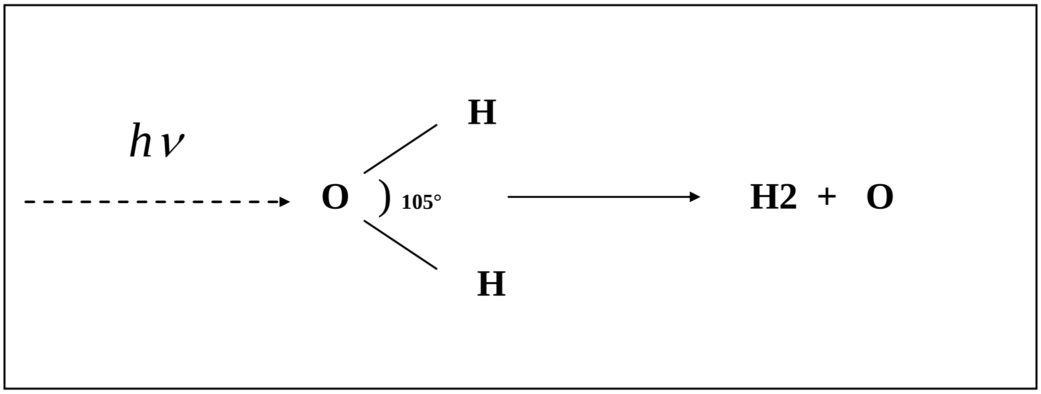

Fig. 2 – Logical scheme of the so-called "Photon-phonon Inter-action" Process

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