

Rational of the Mathematical Formulation of the Max. Em-Power Principle

Such a mathematical formulation was achieved through *four* logical steps. they are Rational of, as illustrated in the two papers presented at the First and Second Emery Conferences, held in Gainesville (Florida, USA), in 1999 and 2001, respectively.

The first step (see “Toward a Mathematical Formulation of the Maximum Em-Power Principle”) consisted in giving the most appropriate formulation of the Principle in *steady-state* conditions. This is because the Principle, according to Odum, has to be considered valid under such conditions and, at most, under very slow transients. That is, the Principle is valid when the considered System has reach its maximum possible organization.

As much more widely illustrated in the paper, such a formulation can be given as follows

$$\sum_{j=1}^m \alpha_j^* \cdot \alpha_j \cdot Em(u_j) + \sum_{k=1}^n \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, \dots, u_m) = \sum_{l=1}^p \beta_l^* \cdot \beta_l \cdot Em(y_l) \quad (3.18)$$

where

- α_j, β_l are the *co-injection* and *co-production* coefficients for each sub-System
- α_j^*, β_l^* are their associated *re-normalization factors* (referred to the Whole System)
- $\Phi_k^*(u_1, u_2, \dots, u_m)$ is the “*equivalent*” Source Term relative to the *k-th* sub-System
- γ_k, γ_k^* are the corresponding “*weight*” and its associated *re-normalization factor*.

In addition, as a second logical step, we gave a preliminary formulation of the Principle in variable conditions:

$$\sum_{j=1}^m \alpha_j^* \cdot \alpha_j \cdot \dot{E} m(u_j) + \sum_{k=1}^n \gamma_k^* \cdot \gamma_k \cdot \dot{\Phi}_k^*(u_1, u_2, \dots, u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \sum_{l=1}^p \beta_l^* \cdot \beta_l \cdot \dot{E} m(y_l) \quad (3.21)$$

where the only “*structural novelty*”, with respect to Eq. (3.18), is represented by the *Global Accumulation Term* $A_{D_s}(t)$. This term (as better illustrated in the paper) does not reduce to a simple “sum” of the *n* sub-System accumulation terms, but is given by an *appropriate sum* of *n* “*equivalent*” accumulation terms, each one (in turn) expressed as a linear combination of all the *n* *real* sub-System contributions.

The third logical step (see “Mathematical Formulation of the Maximum Em-Power Principle”) consisted in analyzing the more appropriate formal structure of Eq. (3.21), by showing that it becomes mostly adherent to the M. Em-P. Principle when formulated as

$$\sum_{k=1}^n \gamma_k^* \cdot \gamma_k \cdot \Phi_k^*(u_1, u_2, \dots, u_m) = \frac{\partial}{\partial t} A_{D_s}(t) + \dot{E} m_{circ} + \sum_{l=1}^p \dot{E} m(y_l) - \sum_{j=1}^m \alpha_j^* \cdot \alpha_j \cdot \dot{E} m(u_j) \quad (2.3),$$

because in this form Eq. (2.3) asserts that: “The System tends to maximize the autogenerated Emery flow (first side) in order to maximize the rate of accumulated Emery, together with circulating and output Emery flows, at net of the equivalent input one (second side)”;

The final step consisted in giving the Mathematical Formulation of the Principle in a more “compact” form, in analogy with the traditional Principles of Thermodynamic, in order to facilitate a successive comparison between the latter and the novelties introduced by the Maximum Em-Power Principle:

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V = \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (4.3),$$

where:

- φ_v^* = the “*equivalent*” Source Term per unit volume
- Γ = the *local structural amplification* and *re-normalization factor* (corresponding to the product of the coefficients γ_k^* and γ_k in the case of discrete form (see Eq. (2.3)), which also accounts for the structural variations with time;

$em_v^* = em_{v,m} + em_{v,q}^+ + em_{v,w}^+$, in which

$em_{v,m} = C \cdot \rho \cdot ex$ is the Energy per unit volume associated to the mass (thus transportable by mass flows)

$em_{v,q}^+$ = the Energy per unit volume associated to heat source terms

$em_{v,w}^+$ = the Energy per unit volume associated to work source terms

and $\forall D^*(t) \subseteq S_U(t)$ indicates that Eq. (4.3) is valid for any Domain (D^*) belonging to Universal Space ($S_U(t)$).

Such a Principle, in the light of the previous considerations, may be verbally formulated as follows: “Every System tends to organize its internal structure to generate progressively increasing spring-Energy levels in order to maximize the flow of processed (or “useful”) Energy”.

Formulation (4.3) clearly expresses a *tendency* Principle. In fact it may be seen as constituted by three parts:

1st part:
$$\frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (5.1)$$

which corresponds to the usual definition of the Principle in terms of phenomenological *effects*: “Every System tends to maximize the flow of processed (or “useful”) Energy”;

2nd part:
$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (5.2)$$

which points out the internal *causes* of such effects: “Every System tends to organize its internal structure for progressively increasing spring-Energy levels”;

3rd part:
$$\int_{D^*(t)} \Gamma \varphi_v^* d_3V \stackrel{\rightarrow}{=} \frac{d}{dt} \int_{D^*(t)} em_v^* d_3V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (5.3)$$

which emphasizes that its mathematical structure is a logical consequence of the first two mentioned parts and points out the existing direct relationship between internal *causes* (5.2) and phenomenological *effects* (5.1).

The symbol $\stackrel{\rightarrow}{=}$ has been specifically introduced to emphasize the *versus* of the equivalence which goes *from causes to effects*.¹ Thus an alternative verbal formulation which is able to include *both* the first and the second side of Eq. (5.3) could be the following one: “Every System tends to maximize its internal structure level in order to maximize its Spring-Energy Flow which, in turn, maximizes the flow of processed Energy”.

Immediately after the presentation of this result at the Second Energy Conference - Gainesville (USA, Florida), September 20-22, 2001- Prof. Odum invited me to transform the paper presented into a book, in which it would be possible to show: i) the presuppositions of such a formulation; ii) the comparison with the other three well-known Principles of Thermodynamics; iii) and thus its real novelty as a “*Fourth*” Thermodynamic Principle.

This novelty is substantially related to the traditional perspective of Science, which has often spoken about “quality”, but the latter has always been understood as being a simple “property”, a “characteristic” or a “specific aspect” of a particular phenomenon or of a physical quantity. On the contrary, the novelty introduced by the Maximum Em-Power Principle is literally *extra-ordinary*: Quality, in fact, continues to be a “property” or a “characteristic” of any physical entity, but it is now understood as being *never reducible* to its phenomenological premises or to our traditional mental categories. This is particularly evident in the definition of Energy concept, fundamentally based on its associated *non-conservative algebraic rules*, especially when referred to Generative Processes and their corresponding Energy accounting techniques.

This is why the book, published in November 2002, was entitled “*The Maximum Em-Power Principle as the basis for a Thermodynamic of Quality*”.

Prof. Odum, before his premature death (September 11, 2002), had the opportunity to read the draft version of the book at the end of July 2002 and conveyed me his appreciation of the same by letter. In this respect, I would like to conclude this section by quoting some lines from Prof. Mark Brown’ preface to the book:

¹ Even if such a symbol is not adopted in writing the following equations, it will always be understood as being substituted by the *convention* according to which the equations will be (generally) written: the *left* side will represent the *causes* and the *right* side will represent the corresponding *effects*. Such a physical-mathematical convention (*from left to right*) is in some way analogous to the *topological* convention adopted in Energy Analysis System Diagrams.

“Odum agreed to help fund its publication under his long held belief that shared information is the most powerful and lasting form of emergy there is. Unfortunately, Odum, died on September 11, 2002, before the publication of this book. During his last months, we discussed this work, and his feeling was that it represents a major contribution to the field, and that it was a very important next step in the formulation of his life work.”

The advances introduced by this book appear from chapter 5 on, where, by starting from the most updated formulation (see Eq. (5.3)), we developed some theoretical reflections (synthetically reproduced here below), which allowed us to point out “A possible new perspective suggested by the Maximum Em-Power Principle”, which also led us to the development of a more advanced formulation of the same.

Our reflections started from the consideration that the little arrow which appears in the symbol $\overset{\rightarrow}{=}$, adopted for the first time in writing Eq. (5.3), has never been explicitly understood as a form of implication, neither logical nor physical. It only represents, as previously specified, the sense of (logical and temporal) *priority* between “causes” and “effects”.

It however comes spontaneously to ask: if there are “causes” and “effects”, what sort of causality are we talking (and thinking) about? In fact:

- A) When the M. Em-P. Principle is considered in its comprehensive formulation, that is as a *tendency* Principle, it simply “records” (from a phenomenological point of view) and “reflects” (in its mathematical structure) a characteristic *pro-pension* of any System toward a certain form of self-organization. Such a pro-pension, however, is a sort of *specific tension* due to its *internal* Emergy sources. This fact could induce us to think of a certain form of intentionality, but it would be better for this aspect to be termed as *in-tension-ality*: that is a sort of internal tension that could even be considered as an interior tension;
- B) When the mathematical formulation of the Principle is considered in its organic constitution, that is as being made up of two distinct deeply related parts, the sign of equality necessarily implies a strict equality between the pertinent quantities in both sides.
This fact, however, does not necessarily imply a corresponding identity between the pertinent *qualities* expressed by the two sides respectively. In fact there might be an increase in Quality by passing from “causes” (left side) to corresponding “effects” (right side);
- C) In this sense the little arrow which is present in the symbol $\overset{\rightarrow}{=}$ should remind us of the versus according to which we see the emerging of variety, differences, novelties: in any case an increase in Quality;
- D) The same symbol, which has been understood (up to now) as representing only the sense of priority between “causes” and “effects”, can thus also be considered, from now on, as indicating the sense of a directional increasing in Quality (understood, at least, as the result of a “spring” generative process);
- E) The same symbol, if interpreted as an indicator of a possible sense of “implication”, seems to suggest not specifically the traditional concept of effectiveness (as a result of a necessary causality) but the softer concept of efficacy (understood as the result of a “spring” causality);
- F) In such a perspective the traditional derivative which appears in Eq. (5.3) seems not to be sufficiently adequate to express such a “variation” in Quality, in addition to the habitual variation in quantity;
In fact, from a linguistic-mathematical point of view, the traditional derivative is defined according to an a posteriori perspective. It is thus simply able to record the quantity variations which have already (and just) happened and that can be recognized in the final effects of the process.
Consequently, the attempt at also describing the emerging Quality suggests we introduce a different concept of derivative, able to represent both quality and quantity variations: **The “Incipient” Derivative**
- G) This latter consideration implies that Eq. (5.3) should be thought of as better written as follows

$$\int_{D^*(t)} \Gamma \phi_v^* d_3 V \overset{\rightarrow}{=} \frac{\tilde{d}}{\tilde{d}t} \int em_v^* d_3 V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (5.11)$$

where the symbol $\frac{\tilde{d}}{\tilde{d}t}$ represents the already mentioned “incipient” derivative.

Such a new formulation does not (apparently) change the quantitative aspects pertaining to the M. Em-P. Principle as formulated in Eq. (5.3), because the formulation under consideration only makes use of the first order derivative (in fact important quantitative variations may appear only in the presence of any order fractional derivatives or higher order integer derivatives. In reality, such a passage from an a posteriori mathematical description to a mathematical description conceived of as being “incipient” (that is capable of

“following the emerging Quality in its continuous process of genesis” and here simply anticipated) represents one of the most important aspects concerning the mathematical formulation of the M. Em-P. Principle.

Its decisive importance will be widely illustrated and sustained on the basis of some applications, especially those concerning unexplained problems in Physics, Biology, and Economics, which will however be solved by means of such a new concept of derivative;

H) The successful mathematical description in terms of “incipient” derivatives will also suggest we replace the concept of logical coherence with that of logical adherence.

This fact will contribute to the introduction of profound novelties in the interpretation of logical “necessity” as well as “physical” necessity;

J) This also suggests we use, when necessary, a slightly different symbology to clearly point out such an aspect of adherence on behalf of the “effects” to their pertinent “generative causes”. In such a case we will then re-write Eq. (5.11) as follows

$$\int_{D^*(t)} \Gamma \varphi_v^* d_3 V \stackrel{[\rightarrow]}{\sim} \frac{d}{dt} \int_{D^*(t)} em_v^* d_3 V \rightarrow Max \quad , \quad \forall D^*(t) \subseteq S_U(t) \quad (5.12);$$

where the symbol “[\rightarrow]” represents either a logical consequence (second side) adherent to its premises (first side) or physical effects (second side) adherent to their “spring” causes (first side) or both;

K) Finally, the M. Em-P. Principle introduces some other profound differences with respect to the traditional Thermodynamic Principle not only when it is formulated in “positive” terms (such as in Eq. (5.3)), but also when its formulation is correspondently given in “negative terms”.

In fact the aspect of a possible “negative formulation” can be surely retained for the M. Em-P. Principle too, but its meaning radically changes with respect to the corresponding “negative” formulation of the Classical Thermodynamic Principles.

All such aspects, although only delineated (or simply enunciated), should be sufficient to understand that the Maximum Em-Power Principle represents a significant novelty in the panorama of Classical Thermodynamics and its mathematical formulation clearly enhances such a characteristics by bringing out all its intrinsic innovative potentiality.

All the various above-mentioned aspects, here simply recalled, gave origin to a process on mathematical developments (lasted about eight years) which could be termed as

“The passage from the Maximum Em-Power Principle to the Maximum Ordinality Principle”