12

The Relevance of Emerging Solutions for Thinking, Decision Making and Acting
The case of Smart Grids

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ABSTRACT

The paper presents a real novelty in Mathematics, which may have an enormous relevance in our way of Thinking, Decision Making and Acting.

At the same time, the paper would like to represent an explicit "Tribute" to Prof. Odum, because the original concept is already seminally present in his well-known Rules of Emergy Algebra.

This mathematical novelty is represented by the so-called "Emerging Solutions", which are radically different from solutions to traditional mathematical problems. This is because any traditional solution to an algebraic or differential problem is always represented by a formal expression that, when reintroduced into the initial formulation of the problem, reduces the latter to a perfect identity.

Emerging Solutions, on the contrary, show an Ordinal Information content which is always much higher than the corresponding content pertaining to the initial formulation of the Problem. Emerging Solutions, in fact, originate from any physical problem when this is formulated in accordance with the Maximum Ordinality Principle, and thus understood in Ordinal Terms.

Such a property, which represents one of the most interesting aspects of the of Maximum Ordinality Principle, then suggests we adopt a Generative way of Thinking when designing a new practical application. The same happens at the level of Will, that is at the level of Decision Making. Obviously, if we really want to take advantage of those "Emerging Exits" which arise from the physical behavior of the system. Finally, at the level of Acting, if we are really interested in favoring any "emerging behavior" of the system which is decisively capable to improve our design. For instance, to get the maximum intrinsic stability of the system, so as to prevent any possible disturbance that might significantly alter its expected behavior.

All these aspects will be illustrated through the case of Smart Grids, with particular reference to their large scale "intrinsic" instability and their recognized strong vulnerability to "cyber" attacks.

INTRODUCTION

The paper is substantially aimed at presenting a real novelty in Mathematics (understood as a "Formal Language"), that is the "Emerging Solutions". These represent one of the most interesting aspects of the Maximum Ordinality Principle which, on the other hand, is nothing but the reformulation of the Maximum Em-Power Principle once "deprived" of any reference to Classical Thermodynamics (such as Energy, Exergy, and so on). In this sense, the paper would also like to represent an explicit "Tribute" to Prof. Odum, because the concept of "Emerging Solutions" is already seminally present in his well-known Rules of Emergy Algebra (Brown and Herendeen 1996).

In the second part of the paper we will present some Ostensive Examples of "Emerging Solutions" (thus referred to Mathematical Models) together with their correspondence to some "Emerging Phenomena", which can be understood as "Emerging Exits" from Generative Processes.

All these examples will then converge to show, in the third part of the paper, analogous aspects in the study case of Smart Grids.

Finally, the comparison between "Emerging Solutions" and "Emerging Exits" from Generative Processes will bring out the potential relevance of the former, in terms of Thinking, Decision Making and Acting, when designing a new practical application.

More precisely, the case of Smart Grids will show how the concept of "Emerging Solutions" is able to suggest the best strategies to optimize the System. In fact, because of their capability of describing foreseeable behaviors of the System, they "reveal" how to improve the well-known large scale "intrinsic" instability of Smart Grids and their recognized strong vulnerability to "cyber" attacks.

THE CONCEPT OF EMERGING SOLUTIONS

The simplest way of presenting "Emerging Solutions" is that of making a comparison with traditional solutions, as synoptically shown in Table 1. Nonetheless, from a conceptual point of view, Emerging Solutions can be termed as such precisely because *they always show an Ordinal Information content which is much higher than the corresponding content of the initial formulation of the Problem*.

Table 1 - Traditional Solutions vs "Emerging Solutions"

Traditional Solutions (TS)	**"Emerging Solutions" (ES)**
1) TS are those solutions which originate from any traditional algebraic or differential problem	1') ES are those Solutions which originate from any Ordinal Differential Problem formulated in terms of "incipient" derivatives
2) Consequently, they are those solutions which describe any system when modeled in terms of traditional physical Laws	2') More specifically, they are those Solutions which describe any System modeled according to the Maximum Ordinality Principle (M.O.P.)
3) TS are always represented by a formal expression that, when reintroduced into the initial formulation of the problem, reduces the latter to a perfect identity	3') ES progressively acquire their increasing Ordinality during the same solution process, so that, if reintroduced into the initial formulation of the Problem, the latter *does not reduce* to a perfect identity
4) They are "solution" to a problem in the sense of "loosing a *knot* "	4') They are "solution" to an Ordinal Problem in the sense of "disclosing *a seed* "

As an introductory Ostensive Example it is worth mentioning the case of the "Three-body Problem". In fact, if the formal Solution to this problem, obtained in terms of the M.O.P., is re-introduced into the initial formulation of the same, we get the formulation of a new Ordinal Problem, corresponding to a "Six-body Problem" (and so on). This is because the "Inter-action" between the higher Ordinal information content of the Solution and the lower information content of the initial Problem give rise to a sort of a Feed-Back of Ordinal Nature, which can evidently be seen as a generalized version of the well-known Feed-Back in Emergy Algebra. This is precisely because the concept of “Emerging Solutions” traces back to the same Rules of Emergy Algebra.

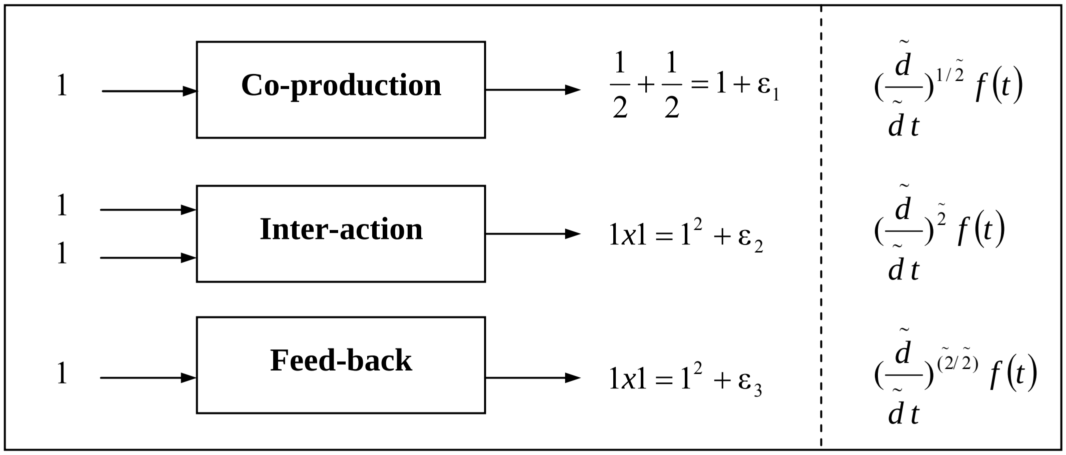

Fig. 1 - Transformity as a “cipher” of the internal self-organizing activity of the System and corresponding representation under Dynamic Conditions in terms of “incipient” derivatives

If we consider in fact the Rules of Emergy Algebra pertaining to the three fundamental Processes (Co-production, Inter-action, Feed-back) schematically represented in Fig. 1, we can easily recognize that the *non-conservative* Algebra adopted substantially asserts that: i) “1 + 1 = 2 + something else” (in a Co-production); ii) whereas “1 times 1 = 1 + something extra”, where this “extra” strictly depends on the nature of the Process (Inter-action or Feed-back, respectively). In this sense Transformity may (also) be interpreted as a “cipher” of the internal self-organizing “activity” of the System (where the term “cipher” is here understood in a gnosiological sense). It would thus indicate that: *there are processes, in Nature, which cannot be considered as being pure* “*mechanisms*”.

In other words they are not describable in mere functional terms, because their outputs show an unexpected “excess” with respect to their pertinent inputs. Such an “excess” can be termed as *Quality* (with a capital Q) exactly because it is no longer understood as a simple “property” or a “characteristic” of a given phenomenon, but as being any *emerging* “property” (from the considered Process) *never reducible* to its phenomenological premises or to our traditional mental categories (Giannantoni 2009, 2010a; see also Anderson 1972).

This is precisely the sense of Transformity when this is understood as a “cipher” of the internal self-organizing “activity” of the System. This is also the reason why, according to the convention adopted for the term *Quality*, all the fundamental terms referable to such a concept are usually *capitalized* in this paper, so as to systematically remind their fundamental relationship to that concept.

It is then evident that, when transforming such a non-conservative Algebra (which is valid in steady-state conditions) to *dynamic conditions*, we end up by introducing a corresponding *non-conservative* Differential Calculus (the transpositions of the Rules in terms of “incipient” derivatives are shown in Fig. 2). This is precisely because the traditional derivative is not properly apt to represent such a concept of “cipher”.

EMERGING SOLUTIONS FROM THE MAX. ORDINALITY PRINCIPLE

The Maximum Ordinality Principle (Giannantoni 2010a) is nothing but the reformulation of the Maximum Em-Power Principle (Odum 1994a,b,c) given in a more general form by means of a new concept of derivative, the “incipient” derivative, whose mathematical definition has already been presented in (Giannantoni 2001a, 2002, 2004, 2008, 2009b, 2010a). In this way both Emergy and

Transformity are replaced by the concept of Ordinality. This is why the principle was renamed as the Maximum Ordinality Principle (Giannantoni 2010a,b). Its corresponding enunciation then becomes: "Every System tends to Maximize its own Ordinality, including that of the surrounding habitat". In formal terms

$$(\tilde{d}/\tilde{d}\,t)^{(\tilde{m}/\tilde{n})}\{\tilde{r}\}_s = 0 \qquad\qquad (\tilde{m}/\tilde{n}) \rightarrow Max \tag{2}$$

where: $(\tilde{d}/\tilde{d}\,t)$ is the symbol of the incipient derivative; $(\tilde{m}/\tilde{n})$ is the Ordinality of the System, which represents the Structural Organization of the same in terms of Co-Productions, Inter-Actions, Feed-Backs; while $\{\tilde{r}\}_s$ is the proper Space of the System.

Equation (2), considered together with its associated initial conditions, leads to the following explicit Solution

$$\{\tilde{r}\}_s = e^{\begin{Bmatrix} \tilde{\alpha}_{11}(t) & \tilde{\alpha}_{12}(t) & \dots & \tilde{\alpha}_{1n}(t) \\ \tilde{\alpha}_{21}(t) & \tilde{\alpha}_{22}(t) & \dots & \tilde{\alpha}_{2n}(t) \\ \dots & \dots & \dots & \dots \\ \tilde{\alpha}_{m1}(t) & \tilde{\alpha}_{m2}(t) & \dots & \tilde{\alpha}_{mn}(t) \end{Bmatrix}} \tag{3}$$

Strictly speaking, enunciation (2) formally asserts that: "every System is a Self-organizing System which persistently propends toward the Maximum Ordinality conditions". However, when a Self-organizing System *effectively reaches* such very special conditions, it presents itself as being self-structured in a radically different way with respect to its initial Ordinality. This is because the latter has evolved according to the following Trans-formation

$$(\tilde{m}/\tilde{n}) \rightarrow \{\{\tilde{2}/\tilde{2}\} \uparrow \{\tilde{2} \uparrow\}\} \uparrow \tilde{N} \tag{4}$$

where: $\{\tilde{2}/\tilde{2}\}$ represents a "binary-duet" coupling; the Ordinal power $\{\tilde{2} \uparrow\}$ indicates the "perfect specularity" of the previous "binary-duet" structure; while $\uparrow \tilde{N}$ indicates the Ordinal Over-structure of the $\tilde{N}$ elements of the System considered as a Whole (this is the reason for the "tilde" notation) (see Giannantoni 2009, 2010a,b). More precisely, Trans-formation (4) is due to the correlative circumstance that, under Maximum Ordinality conditions, the System also achieves its *Maximum Internal Stability*.

This corresponds to the fact that *each single couple* of elements structures itself in a "binary-duet" relationship, according to the evolution described by the following equation

$$(\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\{\tilde{r}\} \otimes (\tilde{d}/\tilde{d}\,t)^{(\tilde{2}/\tilde{2})}\{\tilde{r}\}\} = 0 \tag{5}$$

Consequently, while the genesis of the Ordinal Structure of the System (expressed by solution (3)) is only due to Eq. (2), the generating Process of the internal Ordinal Stability of the System is expresses by Eq. (5), which, for the sake of simplicity and clarity, has been formulated with reference to any *single couple* of elements that progressively structure in a "binary-duet" Relationship.

Equation (5) formally asserts that the proper Space of the System (now considered as being made up of two sole elements) is coupled with its specific Generativity in such a way as to originate a comprehensive Generative Capacity which is always in equilibrium.[1] Such an equation is precisely that which leads to the afore-mentioned *perfect specularity* which, in the case of two sole elements, is

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[1] The symbol $\otimes$ represents a more general form of the "vector" product. However, in this specific context, it can be assumed as being perfectly equivalent to the traditional vector product.

represented by the Ordinal structure $\{\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{2}\uparrow\}\}$, while in the case of $\tilde{N}$ elements is represented by the right hand side of Eq. (4).

Under these conditions the solution to Eq. (2) (and associated Eq. (5)) can be expressed in the form of a new exponential Ordinal Matrix

$$\{\tilde{r}\}_s = e^{\left\{\begin{matrix} 0 & \tilde{\alpha}_{12}(t) & ... & \tilde{\alpha}_{1N}(t) \\ \tilde{\alpha}_{21}(t) & 0 & ... & \tilde{\alpha}_{2N}(t) \\ ... & ... & ... & ... \\ \tilde{\alpha}_{N1}(t) & \tilde{\alpha}_{N2}(t) & ... & 0 \end{matrix}\right\}} \qquad (6),$$

where all the elements of the main diagonal are equal to zero, whereas all the other elements $\tilde{\alpha}_{ij}(t)$ satisfy the following *specularity* Relationships

$$\{\tilde{\alpha}_{ij}(t)\}^{\{\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{2}\uparrow\}\}} \overset{*}{=} \{\tilde{\alpha}_{ji}(t)\}^{\{\{\tilde{2}/\tilde{2}\}\uparrow\{\tilde{2}\uparrow\}\}} \qquad (7),$$

which represent a much more profound concept than the traditional symmetry (correspondingly, the symbol "$\overset{*}{=}$" indicates a simple assignation condition).

Moreover, the Generative Process that leads the System to its Maximum Ordinality and, at the same time, to its Maximum Stability conditions, also restructures the internal relationships between the various elements in such way as these show an additional "*emerging*" *property*, expressed by the following Harmony Relationships:

$$\tilde{\lambda}_{12} \oplus \tilde{\alpha}_{12}(t) \overset{*}{=} \tilde{\lambda}_{1j} \oplus \tilde{\alpha}_{1j}(t) \qquad \text{for } j = 3,4,......N \qquad (8)$$

together with all their associated *incipient* derivatives, up to the order N-1

$$\{\overset{\circ}{\tilde{\lambda}}_{12} \oplus \overset{\circ}{\tilde{\alpha}}_{12}(t)\}^{\tilde{k}} \overset{*}{=} \{\overset{\circ}{\tilde{\lambda}}_{1j} \oplus \overset{\circ}{\tilde{\alpha}}_{1j}(t)\}^{\tilde{k}} \qquad \text{for } k = 1,2,......N-1 \qquad (9),$$

where $\tilde{\lambda}_{ij}$ represent their corresponding *internal reciprocal* Correlating Factors.

This means that all the elements of Ordinal Matrix (6) can be obtained on the basis of 1 sole couple $\tilde{\alpha}_{ij}(t)$ and N-1 associated Correlating Factors.

As a consequence of all the these successive formal passages the Solution to the System, originally structured in form (3), ends up by *progressively* assuming the following more general "emerging" form

$$\{\tilde{r}\}_s = e^{\left\{\begin{matrix} 0 & \tilde{\lambda}_{12}(t)\oplus\tilde{\alpha}_{12}(t) & ... & \tilde{\lambda}_{12}(t)\oplus\tilde{\alpha}_{1N}(t) \\ \tilde{\lambda}_{21}(t)\oplus\tilde{\alpha}_{21}(t) & 0 & ... & \tilde{\lambda}_{2N}(t)\oplus\tilde{\alpha}_{2N}(t) \\ ... & ... & ... & ... \\ \tilde{\lambda}_{N1}(t)\oplus\tilde{\alpha}_{N1}(t) & \tilde{\lambda}_{N2}(t)\oplus\tilde{\alpha}_{N2}(t) & ... & 0 \end{matrix}\right\}} \qquad (10).$$

In this respect, it is worth pointing out more clearly such a sequence of progressively "Emerging Solutions": i) from initial form (3), which characterizes the System in its natural tendency toward the Maximum Ordinality Conditions; ii) we pass to Solution (6) which expresses that, when the System effectively reaches such special conditions, it has also achieved its Maximum Stability conditions (Eqs. (7)), through a Generating Process described by Eq. (5); iii) the latter Process, however, does not characterize the sole single couples $\tilde{\alpha}_{ij}(t)$ (see Eq. (8)), but also all their associated "incipient" derivatives (see Eqs. (9)); iv) the latter are exactly those Ordinal Relationships that lead to final Solution (10), which shows that the System is also characterized by an additional "emerging" property: its internal *Ordinal Harmony*.

FROM EMERGING *SOLUTIONS* TO EMERGING *EXITS*

While "Emerging Solutions" are understood as a *mathematical* concept, the expression "Emerging Exits" specifically refers to those *phenomenological* aspects (or properties) which appear as being *not reducible* to traditional physical Laws.

In this section we will mention three examples of Emerging Solutions that can be obtained by adopting the M.O.P. as a reference modeling criterion and, correspondingly, the associated phenomenological "Emerging Exits" that can be interpreted on the basis of the former.

Table 2 - Mathematical Models, corresponding Emerging Solutions and associated Emerging Exits

Mathematical Models based on the M.O.P.	**Emerging Solutions** (to M.O.P. Mathematical Models)	**Emerging Exits** (phenomenological properties)
1) The Three (N) Body Problem presents an explicit solution	Non-superimposition of Spaces $\{\tilde{r}\}_s \neq \sum \{\tilde{r}\}_i$	Expansion of the Universe $\{\tilde{En}\}_s \neq const$
2) Protein Folding becomes a tractable problem	Harmonic spatial configuration based on the Ordinal roots of Unity	Mono-chirality of Proteins
3) The Three (N) Good Problem presents an explicit solution	Maximum Ordinality is much more than Pareto's optimality	Ordinal Benefits (not accounted for by GDP)

The first Ostensive Example (see Tab. 2) is represented by the well-known "Three-Body Problem", explicitly dealt with in (Giannantoni 2007) and also recalled in (Giannantoni 2008, 2010a,b).

As is well-known, such a problem does not admit an explicit solution, not even in a closed form (as proved by Poincaré in 1888). Vice versa, when modeled on the basis of the M.O.P., it presents an explicit solution which can easily be extended to N bodies (Giannantoni 2010b, 2011a,b). Such a solution presents itself as an "Emerging Solution" because it shows (among other aspects) the inapplicability of superimposition of spaces (see Tab. 2). Such a mathematical property corresponds, from a phenomelogical point of view, not only to the recognized expansion of the Universe but, above all, to non-conservation of Energy (Giannantoni 2010a). A result already anticipated by the same Poincaré, when asserted that "The conservation of Energy is a limitation imposed to *freedom* of complex systems" (Poincaré 1952, p. 133), and also implicitly recognized by modern Astronomy through the "hypothesis" of "Dark Energy".

The second Ostensive Example is represented by Protein Folding, usually considered as being one of the most important "intractable" problems. In fact, although the problem is thought of as being theoretically solvable in principle, the time required in practice to be solved may range from hundreds to some thousands of years, even when run on the most updated computers (1 Petaflop). On the contrary, when model in adherence to the M.O.P, the Problem becomes "tractable", because it presents an explicit solution (Giannantoni 2010a,b, 2011a). This, in turn, leads to an extremely significant reduction of the computation time (from thousand of years to some minutes), even by adopting a common PC, usually characterized by a much lower computation power (about 1 Gigaflop).

In such a case the explicit solution presents itself as an “Emerging Solution” because, among other aspects, is able to give the Harmonic spatial configuration of the folded Protein, whose topology is described by the Ordinal roots of Unity (sea later on). What’s more, this “Emerging Solution” also paves the way to recognize that the well-known *mono-chirality* of Proteins is an “Emerging Exit” of their Generative Folding Process (Giannantoni 2007, ch. 18, 2010b). A phenomenological property which, as is well-known, still remains unexplained from its first discovery (in the early 1930s), precisely because it results as being *not reducible* to traditional categories and consequential theoretical approaches.

The Third Ostensive Example is represented by the well-known “Three Good Two Factor Problem”, a famous problem of Neo-Classical Economics which does not admit an explicit solution, not even in a closed form. Vice versa, when it is extended to the case of “N-Good Three Factor Problem” (Giannantoni 2011b), also this Problem presents an explicit Solution. The latter, in turn, can still be considered as an “Emerging Solution” because it shows that, from a mathematical point of view, Maximum Economic Ordinality represents something “more” than the traditional concept of Pareto’s optimality. Such an “Emerging Solution” then enables us to recognize, as corresponding “Emerging Exits”, the existence of Ordinal Benefits which, on the contrary, are never accounted for by traditional GDP. This happens not only in steady state conditions, as already shown on the basis of the same Maximum Em-Power Principle (Giannantoni 2009b, Giannantoni & Zoli 2010c), but also under dynamic conditions, on the basis of the Maximum Ordinality Principle (Giannantoni 2011b).

This latter example is also particular apt to show the Relevance of Emerging Solutions for Thinking, Decision Making and Acting with specific reference to the case study of “Smart Grids”.

To this purpose it is worth recalling some basic aspects pertaining to the possibility of a direct transposition of Emerging Solutions between two (or ore) different spaces of analysis.

TRANSPOSITION OF MODELS AMONG PROPER SPACES OF ANALYSYS

The search for solution (10), *both in Classical and Quantum Mechanics* (think of “The N-Body Problem” and “Protein Folding”, respectively) is facilitated not only by the structure of Eqs. (2) and (4), but also (and especially) by the conception of the basic reference space $\{\tilde{r}\}$, which is understood as *one sole entity*. This is why it can more appropriately be represented as follows

$$\{\tilde{r}\} = \{\tilde{x} ® \vec{i} \oplus \tilde{y} ® \vec{j} \oplus \tilde{z} ® \vec{k}\} \qquad (11),$$

where the coordinates $(\tilde{x},\ \tilde{y},\ \tilde{z})$ are understood as representing the exit of a Generative Process (this is the reason for the tilde notation) and the symbols $\oplus$ and ® express more intimate relationships between the same: both in terms of sum ($\oplus$) and in terms of (relational) product (®) (see later on) with respect to the traditional versors $\vec{i}$, $\vec{j}$, $\vec{k}$. However, for practical purposes, it is more useful to adopt the representation obtainable from a generalized version of Moivre’s formula

$$\{\tilde{r}\}_s = \{\tilde{\rho} ® \tilde{i} ® e^{\tilde{\varphi} ® \tilde{j}} ® e^{\tilde{\vartheta} ® \tilde{k}}\} \qquad (12),$$

where the coordinates $\{\tilde{\rho},\ \tilde{\varphi},\ \tilde{\vartheta}\}$ are still considered as being representing the exit of a Generative Process, whereas the traditional $\vec{i}$, $\vec{j}$, $\vec{k}$ are now replaced by three unit spinors $\tilde{i},\ \tilde{j},\ \tilde{k}$, which are defined in such a way as to satisfy the following Relational Product Rules:

$$\tilde{i} ® \tilde{i} = \oplus 1 \qquad \tilde{i} ® \tilde{j} = \tilde{j} \qquad \tilde{i} ® \tilde{k} = \tilde{k} \qquad (13.1)$$

$$\tilde{j} ® \tilde{i} = \tilde{j} \qquad \tilde{j} ® \tilde{j} = \Theta 1 \qquad \tilde{j} ® \tilde{k} = \tilde{k} \qquad (13.2)$$

$$\tilde{k} ® \tilde{i} = \tilde{k} \qquad \tilde{k} ® \tilde{j} = \tilde{k} \qquad \tilde{k} ® \tilde{k} = \Theta 1 \qquad (13.3).$$

Representation (12) is similar (albeit not strictly equivalent) to a system of three complex numbers, characterized by one real unit $(\tilde{i})$ and two imaginary units ($\tilde{j}$ and $\tilde{k}$).

Transposition of the "N body Problem" to the "N good Problem"

The proper Space of the System $\{\tilde{r}\}_s = \{\tilde{\rho}, \tilde{\varphi}, \tilde{\vartheta}\}$, which is typical of both Classical and Quantum Mechanics, can easily be transposed to *Economics* by simply adopting three new variables $\{\tilde{K}, \tilde{L}, \tilde{N}\}$, where $\tilde{K}$ = Capital, $\tilde{L}$ = Labor and $\tilde{N}$ stands for "Natural Resources". In this case, the adoption of "three" productive factors (instead of the traditional "two" factors usually considered in Neo-Classical Economics (NCE), namely Capital and Labor) is not only due to the fact that one of the major criticisms addressed to NCE is that of neglecting Nature as the *third fundamental factor* and, consequently, the *intrinsic value* of Natural Resources. It is especially due to the fact that *three* distinct variables enable us to represent, through their Ordinal Relationships, the three fundamental Processes pointed out by Prof. Odum (Co-Production, Inter-Action, Feed-Back).

Under such conditions the transposition of the concepts to *Economics* clearly shows that any Good (*i*), represented in the Space of Goods as $\{\tilde{r}\}_{G,i} = \{\tilde{K}_i, \tilde{L}_i, \tilde{N}_i\}$ (14), constitutes *one sole entity* and, at the same time, represents something "*extra*" with respect to the simple "sum" of its factors. This evidently reflects the *Holistic* Approach subjacent to the Maximum Ordinality Principle, which is ever-present in all the developments presented in this paper .

On the basis of such a transposition, the "N good Problem" can still be formulated in terms of the Maximum Ordinality Principle (see Eq. (2)), in order to obtain the explicit general solution (10), which is now understood in the corresponding proper Space of Goods $\{\tilde{r}\}_G = \{\tilde{K}, \tilde{L}, \tilde{N}\}$ (see (14)).

If we now consider *N* different goods, characterized by the arbitrary values $\{\tilde{K}_i, \tilde{L}_i, \tilde{N}_i\}$, there is no certainty that (at least in principle) their "coordinates" satisfy all the afore-mentioned Harmony conditions. Vice versa, when the production of *N* Goods is more appropriately considered as an "Emerging Exit" of Generative Processes, it represents a unique Ordinal entity, because the *N* Goods are "harmoniously" coordinated among themselves according to Relationships (8) and (9).

Under these conditions, in fact, any Ordinal set of Goods is not a simple arithmetical "sum" of the same, but gives origin to something "extra": a *unique* and *irreducible* entity.

Such a property appears as being decisively important for any Decision Maker, because it suggests the best strategy that is effectively able to maximize the Ordinality of the Economic System analyzed and, at the same time, to reduce the exploitation of Natural Resources.

In fact, if the considered Economic System is not at its Maximum Stability conditions, in the presence of a *new* good produced the comprehensive System (now made up of N+1 goods) might evolve toward a progressively lower level of Ordinality. This more or less markedly depends on the initial conditions of the new good added. In many cases, however, the System could even became manifestly *unstable*.

If, on the contrary, the System is at its Maximum Ordinal Stability, the new additional good may or may not respect the Harmonic Conditions of the comprehensive System as a Whole. In the former case the Economic System will always *evolve* toward a higher level of Ordinality (and associated new Maximum Stability). In the latter case, the System will always remain *stable* (albeit at a slightly lower level), because of its capacity (as a Whole) of "assimilating" the external "disturbance", as a consequence of the coordinated generation of Goods due to *all* the harmonious Ordinal Inter-Relationships between the *N* preexisting Generative Processes.

Transposition of the same concepts from Economics to Smart Grids

Let us now consider a Smart Grid, understood as a Production System made up of N electrical Generators, characterized by three parameters: V_j = Voltage, I_j = maximum current Intensity, Φ_j = internal impedance Phase (see Fig. 2).

As a consequence of the Energy conservation Principle we usually assume that

$$I_{tot}(t) = \sum_{j=1}^{N} (I_j \sin \omega t) = (\sum_{j=1}^{N} I_j) \cdot \sin \omega t \qquad (15).$$

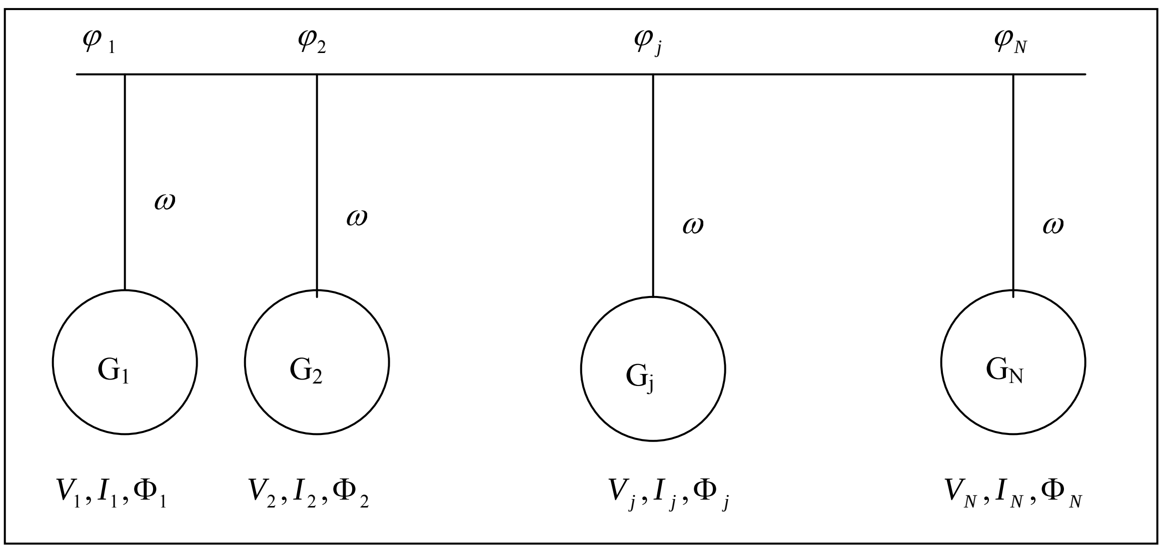

Fig. 2 - A simplified scheme of a Production System made up of N electrical Generators

In reality there is always a distortion "drift", because we are dealing with *N Generative* Processes. Consequently, if we "sum" their corresponding currents, now considered as "Emerging Exits" of the same Processes (thus characterized by the tilde notation), we always have that

$$\{\tilde{I}_{tot}(t)\} = \{\tilde{I}_1(t) \oplus \tilde{I}_2(t) \oplus .....\tilde{I}_n(t)\} \neq (\sum_{j=1}^{N} I_j) \cdot \sin \omega t \qquad (16).$$

Inequality (16) can easily be understood by considering the simple case of two Generators (j and k), and the corresponding "sum" of their corresponding currents, which gives

$$\{\tilde{I}_j(t) \oplus \tilde{I}_k(t)\} \neq I_j(t) + I_k(t) = (I_{j,m} + I_{k,m}) \cdot \sin \omega t \qquad (17).$$

In fact Taylor's expansion series of the function on the right hand side of inequality (17), rewritten as

$$f(t_0+\Delta t)=I_j(t)+I_k(t)=e^{\ln[I_j(t)+I_k(t)]}=e^{\alpha(t)} \qquad (18),$$

where $\alpha(t)=\ln\,\{I_j(t)\oplus I_k(t)\}$, gives

$$f(t_0+\Delta t)=e^{\alpha(t_0)}+e^{\alpha(t_0)}\cdot\dot{\alpha}(t_0)\cdot\frac{\Delta t}{1!}+\sum_{k=2}^{n}\left[\frac{d^k e^{\alpha(t)}}{dt^k}\right]_{t_0}\frac{\Delta t^k}{k!}+....=(I_{j,m}+I_{k,m})\cdot\sin\omega t \quad (19),$$

which (as is well known) represents a perfect sinusoidal trend.

Vice versa, if we consider the left hand side of inequality (17), where the "sum" refers to the same two currents understood as the corresponding Exits of two Generative Processes, we have an Ordinal Relationship, which can analogously be rewritten as

$$\tilde{f}(t_0+\Delta t)=\{\tilde{I}_j(t)\oplus\tilde{I}_k(t)\}=e^{\ln\{\tilde{I}_j(t)\oplus\tilde{I}_k(t)\}}=e^{\tilde{\alpha}(t)} \qquad (20),$$

where $\tilde{\alpha}(t)=\ln\,\{\tilde{I}_j(t)\oplus\tilde{I}_k(t)\}$. In this case, "incipient" Taylor's expansion series of (20), which is namely obtained in terms of incipient derivatives (Giannantoni 2010a), gives

$$\tilde{f}(t_0+\Delta t)=e^{\alpha(t_0)}+e^{\alpha(t_0)}\cdot\overset{\circ}{\alpha}(t_0)\cdot\frac{\Delta t}{1!}+e^{\alpha(t_0)}\sum_{k=2}^{n}[\overset{\circ}{\alpha}(t_0)]^k\frac{\Delta t^k}{k!}\neq(I_{j,m}+I_{k,m})\cdot\sin\omega t \quad (21),$$

which is substantially different from a sinusoidal trend as a consequence of the differences (that appear for $n\geq 2$) between the corresponding terms of the two expansion series, as shown by Eq. (22) (ib.):

$$\sum_{k=2}^{n}\left[\frac{d^k e^{\alpha(t)}}{dt^k}\right]_{t_0}\cdot\frac{\Delta t^k}{k!}\neq e^{\alpha(t_0)}\sum_{k=2}^{n}[\overset{\circ}{\alpha}(t_0)]^k\cdot\frac{\Delta t^k}{k!} \qquad (22).$$

Such a distortion "drift" (with respect to a perfect sinusoidal trend) tends to amplify even under normal exercise conditions, as a consequence of the different currents produced by the *N* generators, because of as the time differentiated increases (or decreases) in the electrical charges to be supplied.

A "drift" which becomes even more marked in the case of a cyber attack. In fact, among the different forms of attacks (see Harris 2010), we may simply think of the case in which such an external interference not only modifies the intensity of the maximum current of the singe plant involved by the attack but, in a special way, tends to significantly change its phase (up to the opposite phase) (ib.).

On the contrary, if the Smart Grid is designed according to in the M.O.P., the corresponding Maximum Stability conditions can always be assured. This is because the various plants are connected to each other in such a way to satisfy the afore-mentioned Harmony Relationships. The latter, under steady state conditions, assume the following form

$$\{\tilde{V}_{1l},\tilde{I}_{1l}\ \tilde{\Phi}_{1l}\}\circledR(\sqrt[N-1]{\tilde{1}})_l\overset{*}{=}\{\tilde{V}_{12},\tilde{I}_{12}\ \tilde{\Phi}_{12}\} \qquad (23),$$

where $\tilde{V}_{ji}=\tilde{V}_j-\tilde{V}_i\quad,\quad\tilde{I}_{ji}=\tilde{I}_j-\tilde{I}_i\quad,\quad\tilde{\Phi}_{ji}=\tilde{\Phi}_j-\tilde{\Phi}_i$
(24),

while $(\sqrt[N-1]{\tilde{1}})$ represents the N-1 Ordinal Roots of Ordinal Unity ($\tilde{1}$). That is:

$$(\sqrt[N-1]{\tilde{1}})=\{\tilde{\alpha}_l\circledR\tilde{i}\oplus\tilde{\beta}_l\circledR\tilde{j}\oplus\tilde{\gamma}_l\circledR\tilde{k}\}\qquad\text{for l = 1,2,.... N-1} \qquad (25)$$

so that $$\{\tilde{\alpha}_l \circledR \tilde{i} \oplus \tilde{\beta}_l \circledR \tilde{j} \oplus \tilde{\gamma}_l \circledR \tilde{k}\}^{N-1} = \tilde{1} \qquad (26).$$

Conditions (23) can evidently be generalized to any kind of exercise conditions. This means that the considered Smart Grid can always be controlled in such a way as to work at its Maximum Ordinal Stability, not only under normal exercise conditions, but also in the case of cyber attacks.

This is because the adoption of Eq. (23), understood as an "Emerging Solution" to the mathematical model of a Smart Grid based on the M.O.P, is able to foresee the corresponding phenomenological "Exits" of the physical System analyzed. This offers some important advantages:

i) the possibility of optimizing the exercise conditions of any Smart Grids already realized;
ii) the possibility of improving the design of any new Smart Grids to be realized;
iii) in both cases, not only as far as the intrinsic stability of the Grid is concerned, but also (and especially) with reference to external disturbances (such as, for instance, cyber attacks).

CONCLUSIONS

The analysis previously presented points out that the traditional design of Smart Grids and their quantitative optimization (Energy produced, Energy supplied, associated costs, etc.) generally do not satisfy the criterion of their maximum intrinsic stability conditions, in a special way in the case of cyber attacks.

On the contrary, the analysis based on the Maximum Ordinality Principle (and its associated "Emerging Solutions") enables the Decision Maker to recognize in advance those (theoretical) optimal working conditions which realize the Maximum Ordinality of the System and, at the same time, to favor the corresponding "emerging behavior" (as an "Emerging Exit"), which is decisively capable to improve the internal Ordinal Stability of the System with respect to any possible (internal or external) transient conditions.

In other terms, the introduction of the concept of *"Emerging Solutions" suggests :*

i) we Think in Generative terms when designing any new practical application;

ii) we Make Decisions in the respect of those solutions which are "emerging" from the mathematical model, in such a way as to take the maximum advantage from those corresponding "Emerging Exits" which are foreseen to be arising from the physical behavior of the system;

iii) we adopt consequential Actions for favoring that specific "emerging behavior" of the System which appears as being decisively capable to improve our design.

In the particular case of Smart Grids, in order to get the Maximum Intrinsic Stability of the System, so as to prevent any possible disturbance that might significantly alter its expected behavior.

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www.ordinality.org: author's website that presents a general framework about the M.O.P, from the Mathematical Formulation of the Maximum Em-Power Principle up to the Mathematical Formulation of the M.O.P., together with the various Ostensive Examples mentioned in this paper.