The Three-body Problem

The Three-body Problem was proved to be *intrinsically unsolvable* in Classical Mechanics by Poincaré (1887). The Problem in fact it is described by an 18th-order system of ordinary differential equations which, however, admits only 2 first order *closed form* integrals (energy and areas) (Poincaré, 1899, vol. 1, p. 253) .[1]

Vice versa, when faced in terms of incipient derivatives, the problem becomes perfectly solvable, in the sense that:

i) there exists *at least* one solution in *a closed form*, as explicitly desired by Poincaré (ib.);
ii) such a solution, in addition, can be obtained (always in *a closed form*) at different hierarchical levels of Ordinality, according to the initial model adopted (Giannantoni 2007, pp. 49-60):

a) as a System made up of three *distinct* bodies
b) as a System made up of three "*binary-duet* " sub-systems
c) as one sole "*ternary* " System made up of three "*binary-duet* " sub-systems;

iii) these solutions, however, are still affected by a form of "drift" related to the supposed independence of the space variables (x,y,z) from each other. A "supposed independence" which is perfectly conform to the aprioristic assumption that the proper Space of the System is of a Euclidean nature.

If, vice versa, the proper space of the System is considered as *one sole thing*, in which the three coordinates (x,y,z) are so strictly related to each other so as to form one sole entity of Ordinal nature (ib.), the problem admits an extremely elegant solution in explicit terms in exponential form (Giannantoni 2008, p. 113).

In such a case, the cardinal structure of the System is nothing but the formal reflex of its Ordinal nature (ib.).

The N-body Problem

The results obtained in the case of "The Three-body Problem" can easily be generalized to "The N-body Problem" [2]. Such an extension is made possible by the fact that *any* non-linear differential equation, written in terms of incipient derivatives, can always be transformed into a linear differential equation (Giannantoni 2007, ch. 3).

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[1] To quote the same Poincaré : "…*le problème de trois Corps n'admet pas d'autre intégrale uniforme que celle des force vives et des aires.* " (ib.), where the concept of "integral" is not simply understood according to the traditional sense of "solution", but as a "function of solutions" (ib., p. 8) structured in the form $F_i[x_1(t), x_2(t),....x_n(t)] = cost$, where $x_1(t),\ x_2(t),....x_n(t)$ represent the generic unknown variables of the considered problem.

[2] The fact that the "Three-body problem", even in its *most general form*, admits at least *one solution in a closed form* when reformulated in IDC, is substantially due to the *intrinsic* and *specific* properties of the *incipient derivatives*. In fact such a solution can be obtained on the basis of the following : i) the *Fundamental Theorem of the Solving Kernel* (Giannantoni, 1995), which gives the general solution of *any* linear differential equation with variable coefficients in terms of the *sole* Solving Kernel; ii) such a solution, in particular, is already structured in a *closed form* (according to Poincaré's definition) and can directly be transposed to *incipient derivatives* (Giannantoni 2007b, ch. 5); iii) in addition, because the Solving Kernel is generally a *function of a function*, such a *transposition* can be directly obtained by means of *Faà di Bruno's formula* (ib., ch. 3); iv) this in fact, being in turn structured in a *closed form*, can directly be transposed to the derivatives of *functions of a function* when the latter are expressed in incipient terms (the only difference is that, in such a case, there are no longer "partitions" and, consequently, related "sums"); v) finally, any traditional non-linear differential equation in TDC can be transformed into a linear Ordinal differential equation in IDC, with the same methodology as already shown, for example, with reference to Riccati's Equation (Gainesville 2004).
On the other hand, such a general procedure, already adopted in other papers and books (e.g., Giannantoni, 2001, 2004a, 2004c, 2006), is the same which enabled us to sustain the general validity of a Differential Calculus (namely IDC), which contemporaneously operates in terms of Ordinality and cardinality (see Giannantoni 2007, ch. 3).

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